

# Exam #1 Review

Date of Exam: Thursday, September 29

1. This exam addresses the books *Shapes and Designs* and *Covering and Surrounding*. You should review all of the material in these books, including the Mathematical Reflections at the end of each chapter.
2. Look over your course notes and the “Summary of Class” posted on the course website. Be able to solve and discuss all of the material we did in class.
3. Look over all of the homework and the solutions to the homework. Be able to solve and discuss all of these problems.
4. Look over all of the ACE problems, even those I did not assign.
5. Be able to give definitions and examples of different kinds of triangles: acute, right, obtuse, scalene, isosceles, equilateral. Be able to draw a classification diagram.
6. Be able to give definitions and examples of different kinds of quadrilaterals: parallelograms, rhombi, squares, trapezoids, isosceles trapezoids, kites, general quadrilaterals. Be able to draw a classification diagram.
7. Explain how to motivate the interior angle sum of every triangle by tearing and rearranging.
8. Given the interior angle sum of every triangle is 180 degrees, be able to prove that the interior angle sum of any quadrilateral, convex or nonconvex, is 360 degrees, by dividing the interior into two triangles.
9. Be able to prove that the interior angle sum of every convex  $n$ -gon is  $180(n - 2)$ , with two different proofs (drawing all diagonals off of one vertex, or introducing a new point in the interior connected to all vertices). Be able to determine the interior angle sum of nonconvex polygons by dividing them into triangles.
10. Be able to prove in two ways that the exterior angle sum of every polygon is 360 degrees (tracing and turning around the exterior, or relating algebraically to the interior angle sum).
11. Be able to describe the relationships among measures of angles (vertical and adjacent) determined by two crossing lines.

12. Be able to describe and prove the relationships among various angles (corresponding, alternate interior, consecutive or adjacent interior, alternate exterior) formed by a transversal of two parallel lines. In particular, we assume the following theorem to be true: If  $x$  and  $y$  are a pair of corresponding angles in a transversal  $n$  of  $\ell$  and  $m$ , and if  $\ell$  and  $m$  are parallel, then  $x = y$ .
13. Be able to use some given relationships among various angles formed by a transversal of two lines to prove that these lines are parallel. In particular, we assume the following theorem to be true: If  $x$  and  $y$  are a pair of corresponding angles in a transversal  $n$  of  $\ell$  and  $m$ , and if  $x = y$ , then  $\ell$  and  $m$  are parallel.
14. Be able to prove that the sum of the interior angles in every triangle is 180 degrees, using a certain diagram with transversals.
15. Be able to describe symmetries (rotational and reflectional) that certain types of polygons possess.
16. Know that in every triangle, the sum of the lengths of any two sides must be greater than the third side (this is the Triangle Inequality).
17. Know that in every quadrilateral, the sum of the lengths of any three sides must be greater than the fourth side.
18. Understand the concepts of area and perimeter. Be able to explain how to determine the areas and perimeters of various shapes, including irregular ones.
19. Know the formula for the area of a square.
20. Knowing the formula for the area of a square, know and be able to prove the formula for the area of a rectangle, even those with noninteger side lengths, using the “two rectangle puzzle” on the handout.
21. Knowing the formula for the area of a rectangle, know and be able to prove the formula for the area of a triangle, even those with noninteger side lengths. There are three cases. In particular, understand that any side may be chosen to be the base.
22. Knowing the formula for the area of a triangle, know and be able to prove the formula for the area of a parallelogram, even those with noninteger side lengths. In particular, understand that any side may be chosen to be the base.
23. Know and be able to prove the formula for the area of a trapezoid.

24. Know and be able to prove the formula for the area of an equilateral triangle and a regular hexagon.
25. Know and be able to prove the formula for the area of a regular  $n$ -gon with side length  $s$  and apothem  $a$ .
26. Know that the circumference of a circle of radius  $r$  is  $2\pi r$  and describe how this relates to the notion of radian.
27. Be able to motivate why the area of a circle of radius  $r$  is between  $2r^2$  and  $4r^2$ .
28. Be able to motivate the relationship  $A = \frac{1}{2}Cr$  for a circle two ways, one by sectors (wedges) and one by “unwrapping” concentric circles, and then use this to obtain  $A = \pi r^2$ .
29. For rectangles with fixed area, know how to write the perimeter in terms of one side length, and how to determine rectangles with minimum and maximum perimeter. Consider both the case in which the rectangles have integer side lengths as well as the case in which the rectangles need not have integer side length.
30. For rectangles with fixed perimeter, know how to write the area in terms of one side length, and how to determine rectangles with minimum and maximum area. Consider both the case in which the rectangles have integer side lengths as well as the case in which the rectangles need not have integer side length.
31. Be able to change units of measurement by “multiplying by 1s”.
32. Be able to define prisms, both right and oblique. Note that if the pair of parallel congruent bases of a prism are not themselves parallelograms, then there is no other choice of bases, but if the prism is a right or oblique prism over a parallelogram, then any pair of opposite faces can be viewed as bases.
33. Be able to justify by stacking up cubes that the volume of a general prism is the area of its base multiplied by its height,  $BH$ .
34. Be able to know and to prove the formulas for the volume and surface area of a right rectangular prism.
35. Be able to draw nets for prisms and pyramids, and to determine volumes and surface areas for prisms from their nets.

36. Note that you must be careful not to confuse the height  $H$  and base area  $B$  of a prism with the height  $h$  and base  $b$  of one of the two-dimensional faces or bases of the prism.