

Illustration of the Euclidean Algorithm

Description of the Euclidean Algorithm.

Input: Two nonnegative integers a, b , not both zero. Without loss of generality, let's assume $a \geq b$.

Output: The gcd $d = (a, b)$ and two integers x, y such that $ax + by = d$.

Steps:

If $b = 0$, then set $d = a$, $x = 1$, and $y = 0$. Stop.

Otherwise (if $b > 0$) find integers n and r such that $a = nb + r$, $0 \leq r < b$. Carry out the Euclidean Algorithm on the pair of integers b, r to find $d = (b, r)$ and integers x_1, y_1 such that $bx_1 + ry_1 = d$. Set $x = y_1$ and $y = x_1 - ny_1$.

See the next page for an example.

Computing that $(180, 152) = 4$ (read down the first column) and $180(11) + 152(-13) = 4$ (read down the second column).

$$\begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \\
 5 \quad 12 \\
 2 \quad 4 \quad 1 \\
 3 \quad 0 \quad 0
 \end{array}
 \qquad
 \begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \\
 5 \quad 12 \quad 0 \\
 2 \quad 4 \quad 1 \quad 1 \\
 3 \quad 0 \quad 0
 \end{array}$$

$$\begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \\
 5 \quad 12
 \end{array}
 \qquad
 \begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \quad 1 \\
 5 \quad 12 \quad 0 \quad -2 \\
 2 \quad 4 \quad 1 \quad 1 \\
 3 \quad 0 \quad 0
 \end{array}$$

$$\begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \\
 5 \quad 12 \\
 2 \quad 4
 \end{array}
 \qquad
 \begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \quad 1 \quad -2 \\
 5 \quad 12 \quad 0 \quad -2 \\
 2 \quad 4 \quad 1 \quad 1 \\
 3 \quad 0 \quad 0
 \end{array}$$

$$\begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \\
 5 \quad 12 \\
 2 \quad 4 \\
 3 \quad 0
 \end{array}
 \qquad
 \begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \quad 1 \quad -2 \quad 11 \\
 5 \quad 12 \quad 0 \quad -2 \\
 2 \quad 4 \quad 1 \quad 1 \\
 3 \quad 0 \quad 0
 \end{array}$$

$$\begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \\
 5 \quad 12 \\
 2 \quad 4 \\
 3 \quad 0
 \end{array}
 \qquad
 \begin{array}{r}
 180 \\
 152 \\
 1 \quad 28 \quad 1 \quad -2 \quad 11 \\
 5 \quad 12 \quad 0 \quad -2 \\
 2 \quad 4 \quad 1 \quad 1 \\
 3 \quad 0 \quad 0
 \end{array}$$