Quotes and Anecdotes

MA310

Spring 2001

1 Some of my Informal Thoughts about Teaching and Learning Mathematics

1. Don't neglect history. Mathematics is far, far from static. Everything you have studied was invented, discovered, or developed at some point in the past, and sometimes not so long ago! Many have the very strange impression that most of mathematics was finished centuries ago and is now fixed and unchanging. For many, the idea of mathematical research is hard to comprehend, but this should not be so. Mathematics is dynamic, alive, and growing by leaps and bounds!

2. Look for concrete (especially physical) examples, models, or visualizations of mathematical concepts. Conversely, look for the mathematics in everyday objects. For example, what can you say about the lengths of the chimes hanging beneath a xylophone?

3. Be conscious of the difference between skills and concepts. Skills are sometimes more easily learned, practiced, and tested than concepts—hence very tempting to both students and teachers to spend most of their time on—but a math course that degenerates into a sequence of rote skills and manipulation without any real understanding is essentially worthless.

4. Practice asking lots of questions. A good question is worth a lot more than a mediocre fact.

5. Think about analogies with learning in other disciplines—for example, learning to read, learning how to play a musical instrument. In both cases there is an absolute necessity for constant practice, without which it is impossible to attain any level of sophistication or appreciation. This can be hard work! You cannot read Shakespeare if you are still sounding out individual words letter by letter. Also, under no circumstances would I want my first grade child to be taught to read by a teacher who could only read on an elementary school level, or my twelfth grade child to take an English class with a teacher who could only function on a twelfth grade level. The great teachers have a sense of perspective on their subjects which transcends the level on which they are teaching, and students are acutely aware of this.

6. There is a high probability that you will frequently encounter students that are more adept mathematically than you. You must be prepared for this, or you may tragically close doors of opportunity forever for your students.

7. Your love of mathematics should and will undoubtedly extend to activities outside of the classroom. Great musicians love music and play music, even when they are not performing. Great writers do not confine their creativity to an 8-hour day.

8. You must learn how to learn new mathematics on your own, and teach your students this crucial skill.

9. Mathematics is not a linear subject, but parts are intricately intertwined into a complex structure. In school and college, certain strands are extracted and taught, sometimes giving a very misleading view of mathematics as a whole. Think about ways to re-weave the mathematical fabric as you teach.

10. Some of the topics that are studied in a semester took decades, centuries, or even millennia to come to fruition. Is it little wonder that it is hard for students to fully comprehend some of the things we are teaching them the first time (or even the second or third time) that they see them? A good example of this are the concepts of continuity and differentiation in calculus. I recommend that you read about the struggle to try to place the less formal or intuitive views of these concepts on a sound footing. How long did it take?

11. Mathematics is an experimental subject, but the truths discovered are subject to verification in a rigorous way. All too often, the final results are presented in such a way that is cleansed of the explanation of the process of investigation and discovery. This is very sad, for much of value has been lost. One good example of this principle is Archimedes' "Method."

12. High quality work is often hard work. Don't cheat yourself (or others) out of the benefits of significant accomplishment.

There are thousands of books and articles that capture the spirit of mathematics and provide an unimaginable amount of wonderful material. I mention a few of my favorites. There are probably newer editions of all of these.

1. All of the books by Martin Gardner on recreational mathematics and related subjects. Most of this material was first published in the Mathematical Recreations column of *Scientific American*. I grew up on this material starting in elementary school.

2. H. Steinhaus, *Mathematical Snapshots*, Oxford University Press. A gallery of fascinating mathematical snippets.

3. G. Polya, *How to Solve it*, Princeton University Press. My high school teacher showed us a movie by Polya on problem solving that made a deep and long-lasting impression on me.

4. W.W. Rouse Ball and H.S.M. Coxeter, *Mathematical Recreations and Essays*, University of Toronto Press, 1974. I read and reread this book as a high school student.