

Music

Tuning a Piano

A *geometric sequence* is a sequence of the form a, ar, ar^2, ar^3, \dots . That is to say, the first term is some number a , and thereafter each term is obtained from the previous one by multiplying by a specific number r . For example, $15, 45, 135, 405, \dots$ is a geometric sequence.

The 88 keys of a piano are tuned according to a geometric sequence. The note A (designated A4) above middle C (designated C4) has frequency 440 Hz. The C note (C5) above middle C has twice the frequency of middle C. There are 13 notes between C4 and C5, inclusive. Determine the frequencies of these 13 notes, explaining your reasoning. Round your answers to two decimal places.

1. C (C4)
2. C \sharp
3. D
4. D \sharp
5. E
6. F
7. F \sharp
8. G
9. G \sharp
10. A 440 Hz
11. A \sharp
12. B
13. C (C5)

To the human ear, two notes played together tend to sound more harmonious if the ratios of their frequencies is a ratio of two small integers. For each of the fractions below, find a note above C4 so that the ration of the frequency of this note to the frequency of C4 is approximately equal to that fraction:

1. 2/1

2. 3/2

3. 4/3

4. 5/4

5. 5/3

6. 6/5

Here is a website with a virtual keyboard to test notes and chords:
www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/music/piano/index.htm.