

MA 341 Homework #4
Due Monday, October 1, in Class

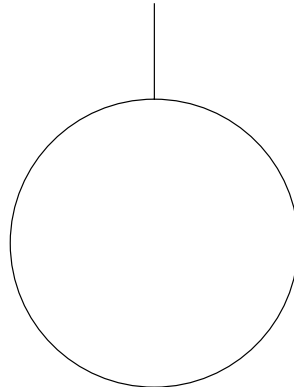
1.

It was the first time that Poole had seen a genuine horizon since he had come to Star City, and it was not quite as far away as he had expected. . . . He used to be good at mental arithmetic—a rare achievement even in his time, and probably much rarer now. The formula to give the horizon distance was a simple one: the square root of twice your height times the radius—the sort of thing you never forgot, even if you wanted to. . .

—Arthur C. Clarke, *3001*, Ballantine Books, New York, 1997, page 71

In the above passage, Frank Poole uses a formula to determine the distance to the horizon given his height above the ground.

- (a) Use algebraic notation to express the formula Poole is using.
- (b) Beginning with the diagram below, derive your own formula. You will need to add some more elements to the diagram.



- (c) Compare your formula to Poole's; you will find that they do not match. How are they different?
- (d) When I was a boy it was possible to see the Atlantic Ocean from the peak of Mt. Washington in New Hampshire. This mountain is 6288 feet high. How far away is the horizon? Express your answer in miles. Assume that the radius of the Earth is 4000 miles. Use both your formula and Poole's formula and comment on the results. Why does Poole's formula work so well, even though it is not correct?

2. Assume we know that the Pythagorean Theorem holds in \mathbf{E}^2 . Consider a third point $C = (x_1, y_2)$, and use this to derive the formula for the distance between the points $A = (x_1, y_1)$ and $B = (x_2, y_2)$. Include a good diagram.
3. Consider the set of all points $P(x, y)$ such that $x^2 - 2x + y^2 - 4y - 4 = 0$. What shape is this set? Provide justification. Why does this make sense? Find a better form of the equation that more clearly represents this set.
4. A camper wants to stop by a river on the way back to her campsite. If the river bank is represented by the line $y = 0$, her present location by the point $A = (0, 2)$, and her campsite by the point $B = (6, 3)$, what is the shortest route she can take? Provide justification. Make a good sketch. It may be helpful to use GeoGebra to experiment.
5. Let L be the line defined by the equation $y = 1$, and let $A = (4, 3)$. Consider the set of all points $P = (x, y)$ such that the distance from P to L equals the distance from P to A . Find an equation to describe this set of points, simplifying it as much as possible. Then use GeoGebra or a similar program to make a good sketch. What kind of shape do you get?
6. Let $A = (-2, 0)$ and $B = (2, 0)$. Consider the set of all points $P = (x, y)$ such that the sum of the distances $PA + PB$ equals 6. Find an equation to describe this set of points, simplifying it as much as possible—in particular, figure out how to get rid of any square roots. Then use GeoGebra or a similar program to make a good sketch. What kind of shape do you get?