

Course Notes

January 18

Today we discussed Section 2.7.1. Next time I hope everyone can take turns presenting solutions to the problems of this section. You may wish to arrange to talk to each other in advance. Problem 1 often appears in MA361. Problem 2 is not complicated, but it is interesting when thinking about infinite sets. Problem 3 is good to think about, because it reinforces the notion and understanding of subgroups. Problem 4, or something like it, often appears in MA361. Problem 5 should not take too long to explain. Problem 6 gets at the heart of the sign of permutations, and why it is well defined. Problem 7 should not take too long to explain, once you know the results of Problem 6. Problem 8 looks more complicated than it is. Problem 9 is a great opportunity for some geometric experimentation.

January 25

Today we discussed the problems in Section 2.7.1, including how to convert permutations in cycle notation into a sequence of transpositions. As an aside we discussed the associativity of the composition of functions. Next time we will discuss Section 2.7.2 and the associated problems. Try making lots of concrete examples as you go through this section. Also, you might consider tackling some of the problems at the same time you are digesting the material. As a bonus to what we did today, you might poke around information on Coxeter groups and Coxeter diagrams, and see what they have to do with thinking about the dihedral groups and the symmetry groups for the tetrahedron and the cube.

February 1

Today we discussed Section 2.7.2, working through Lemma 2.8. Making lots of concrete examples helps! Next week I wish for you to present Theorem 2.9, as well as the solutions to problems 3–5 on page 200. I strongly suggest that you completely write out the solutions to these problems. (We already talked our way through problems 1 and 2, but it wouldn't hurt for you to write down a sketch of these solutions as well.) Also, start digesting the definition of the

cycle index on pages 200–201. Again, concrete examples help a lot.

February 8

Today we completed Theorem 2.9 and the remaining problems in Section 2.7.2, and gained insights that will make the next section very understandable.

February 15

Today we worked through Section 2.7.3 and discussed problems 1–3. After solving problem 2(b) we tried some more cases and made a conjecture for $P_{S_n}(m, \dots, m)$. I believe we should be able to come up with a straightforward verification, using some of the tools from last semester. We also have a conjectured expression for $P_{A_n}(m, \dots, m) - P_{S_n}(m, \dots, m)$. Be sure you write these two conjectures down and think about them. Next time we will finish these problems, and I also urge you to begin to tackle Section 2.7.4.

Some additional notes. Here is a way to think of colorings and S_n : Think of n objects lined up in a row. Color them. Two colorings are equivalent with respect to S_n if you can get from one to the other by simply rearranging the objects. Now for A_n : Two colorings are equivalent with respect to A_n if you can get from one to the other by an even number of swaps (interchanging two items).