

MA/CS415 Homework #5
Due Wednesday, September 29

1. Continue reading and studying pages 30–51.
2. Page 33, #1.
3. Page 37, #6, 10.
4. Page 42, #5.
5. Let G be a simple graph with vertices $\{1, \dots, n\}$. Suppose every edge e of G is assigned a nonnegative cost $c(e)$. Define the cost of any walk to be the sum of the costs of its edges. Define the matrix C with entries

$$C_{ij} = \begin{cases} 0 & \text{if } i = j, \\ c(e) & \text{if } i \neq j, e = \{i, j\} \in E(G), \\ +\infty & \text{if } i \neq j, \{i, j\} \notin E(G). \end{cases}$$

Let $C^{(k)}$ be the matrix obtained by raising C to the k power using weird matrix multiplication. Prove that the ij entry of $C^{(k)}$ is the cost of a minimum cost walk from i to j using *at most* k edges. Suggestion: Refer to the previous homework, and also take into account the effect of the zeroes down the main diagonal.