

**MA 416**  
**Homework #7**  
**Due Monday, April 2**

1. Let  $A$  denote the pay-off matrix for a two-person zero-sum game. This means that if player  $R$  chooses strategy  $i$  and player  $C$  chooses strategy  $j$ , then  $C$  pays  $R$  an amount equal to  $a_{ij}$ .
  - (a) Prove that the linear program associated with  $R$  is the dual of the linear program associated with  $C$ .
  - (b) Use linear programming duality to prove that if the matrix  $A$  is skew-symmetric (i.e.,  $a_{ij} = -a_{ji}$  for all  $i, j$ ), then each of the linear programs yields the same optimal strategy, and each has optimal objective function value 0.
2. Find an optimal strategy for  $R$  for the game of Morra.
3. Find an optimal strategy for  $R$  and an optimal strategy for  $C$  for Morra in the case that  $R$  announces his/her guess first and  $C$  announces his/her guess second. What is average pay-off for this game?