1 The Petro Problem

The Petro Chemical Corporation manufactures two types of chemicals, I and II. One ton of Chemical I requires 1 unit of ingredient A, 2 units of ingredient B, 1 unit of ingredient C, and yields a net profit of 5 dollars. One ton of Chemical II requires 1 unit of A, 1 unit of B, 2 units of C, and yields a net profit of 4 dollars. Available are 70 units of A, 100 units of B, and 120 units of C. The demand is high enough for their products that they know they will sell what they are able to produce.

1. What is the company's optimal production mix?

2. Suppose someone offered to sell Petro one additional unit of A. What is the most that Petro be willing to pay? How many additional units of A would Petro be willing to buy at that price?

3. Suppose someone wished to purchase one unit of A from Petro. What is the minimum price must they offer? How many units of A would Petro be willing sell at that price?

4. Refer to the previous question and answer the analogous questions for B and C.

5. Suppose the profit from Chemical I is no longer \$5. How does the company's optimal production mix change? Answer the analogous question for Chemical II.

Write a report to the manager of Petro expressing the solutions to the above questions in precise but "non-mathematical" language.

2 Lieberknecht Problem

The Lieberknecht family operates a 1000 acre irrigated farm in the Salt River Valley of Arizona. The principal activities are raising wheat, alfalfa, and beef. The Salt Valley Water Authority has just given its water allotments for next year (the Lieberknechts were allotted 2200 acre feet) and the Lieberknechts are busy preparing their production plan for next year. They figure that beef prices will hold at around $600 per ton and wheat will sell at $2.60 per bushel. Best guesses are that they will be able to sell alfalfa at $54 per ton but if they need more alfalfa to feed their beef than they can raise, then they will have to pay $56 per ton to get the alfalfa to their feedlot.

The technological features of the operation are as follows: Wheat yield: 50 bushels per acre; Alfalfa yield: 3 tons per acre.

activity	*labor, machinery and other costs*	*water requirements*	*land requirements*	*alfalfa requirements*
1 acre of wheat	$6	1.5 acre feet	1 acre	
1 acre of alfalfa	8	2.5 acre feet	1 acre	
1 ton of beef	100	.2 acre feet	.11 acre	5 tons

3 MacPhail Problem

Farmer MacPhail has 120 acres which can be used for growing wheat or corn. The yield is 55 bushels per acre per year of wheat or 95 bushels of corn. Any fraction of the 120 acres can be devoted to growing wheat or corn. Labor requirements are 4 hours per acre per year, plus 0.15 hour per bushel of wheat and 0.70 hour per bushel of corn. Cost of seed, fertilizer, etc., is 20 cents per bushel of wheat produced and 12 cents per bushel of corn produced. Wheat can be sold for $1.75 per bushel, and corn for $0.95 per bushel. Wheat can be bought for $2.50 per bushel, and corn for $1.50 per bushel.

In addition, the farmer may raise pigs and/or poultry. The farmer sells the pigs or poultry when they reach the age of one year. A pig sells for $40. He measures the poultry in terms of coops. One coop brings in $40 at the time of sale. One pig requires 25 bushels of wheat or 20 bushels of corn, plus 25 hours of labor and 25 square feet of floor space. One coop of poultry requires 25 bushels of corn or 10 bushels of wheat, plus 40 hours of labor, and 15 square feet of floor space.

The farmer has 10,000 square feet of floor space. He has available per year 2,000 hours of his own time and another 2,000 hours from his family. He can hire labor at $1.50 per hour. However, for each hour of hired labor, 0.15 hour of the farmer's time is required for supervision. How much land should be devoted to corn and how much to wheat, and in addition, how many pigs and/poultry should be raised to maximize the farmer's profits?

4 Radios 'R' Us Problem

The Radios 'R' Us electronics company has a contract to deliver 20,000 radios within the next four weeks. The client is willing to pay \$20 for each radio delivered by the end of the first week, \$18 for those delivered by the end of the second week, \$16 by the end of the third week, and \$14 by the end of the fourth week. Since each worker can assemble only 50 radios per week, the company cannot meet the order with its present labor force of 40; hence it must hire and train temporary help. Any of the experienced workers can be taken off the assembly line to instruct a class of three trainees; after one week of instruction, each of the trainees can either proceed to the assembly line or instruct additional new classes.

At present, the company has no other contracts; hence some workers may become idle once the delivery is completed. All of them, whether permanent or temporary, must be kept on the payroll till the end of the fourth week. The weekly wages of a worker, whether assembling, instructing, or being idle, are \$200; the weekly wages of a trainee are \$100. The production costs, excluding the worker's wages, are \$5 per radio.

5 Kwickie Khem Problem

The Kwickie Khem firm manufactures four products called P_1, P_2, P_3, P_4. Product P_1 can be sold at a profit of \$10 per ton up to a quantity of 10 tons. Quantities of P_1 over 10 tons but not more than 25 tons can be sold at a profit of \$7 per ton. Quantities beyond 25 tons earn a profit of only \$5 per ton.

Product P_2 yields a profit of \$8 per ton up to 7 tons. Quantities of P_2 above 7 tons yield a profit of only \$4 per ton. Everyone who buys P_2 also buys P_4 to go along with it. This is described later. Both products P_1, P_2 can be sold in unlimited amounts.

P_3 is a by-product obtained while producing P_1. Up to 10 tons of P_3 can be sold at \$2 per ton. However, beyond 10 tons there is no market for P_3, and since it can't be stored, it has to be disposed of at a cost to the firm of \$3 per ton.

P_4 is a by-product obtained while producing P_2. Also, P_4 can be produced independently. Every customer who buys θ tons of P_2 has to buy $\theta/2$ tons of P_4 to go along with it for every $\theta \geq 0$. Also, P_4 has an independent market in unlimited quantities. One tone of P_4 yields a profit of \$3 per ton, it it is sold along with P_2. One ton of P_4 sold independently yields a profit of \$2.50 per ton.

Production of one ton of P_1 requires one hour of machine 1 time plus two hours of machine 2 time. One ton of P_2 requires two hours of machine 1 time plus three hours of machine 2 time. Each ton of P_1 produced automatically delivers 3/2 tons of P_3 as a by-product without any additional work. Each ton of P_2 produced yields 1/4 ton of P_4 as a by-product without any additional work. To produce one ton of P_4 independently requires three hours of machine 3 time.

The company has 96 hours of machine 1 time, 120 hours of machine 2 time, and 240 hours of machine 3 time available. The company wishes to maximize its total net profit.

6 Forestry Problem

The setting, New Forest, is a park and forest district of approximately 145 square miles situated in Hampshire, England. Thus we shall count money in pounds and measure lumber in Hoppus feet. (A Hoppus foot, abbreviated h.ft., is the volume of a board 1 foot square and 1 inch thick; close equivalent American measure is the board foot.) The management of New Forest had to choose a felling program for an area of about 30,000 acres, with the objective of maximizing the net discounted revenue over the next decade. The problem considered here involves only part of that area; some 8,500 acres with six different crop types shown in the following table.

Crop Type	Description	Acres	Volume if felled (h.ft./acre)
1	High-volume hardwoods	2,754	2,000
2	Medium-volume hardwoods	850	1,200
3	Low-volume hardwoods	855	700
4	Conifer high forest	1,598	4,000
5	Mixed high forest	405	2,500
6	Bare land	1,761	

The hardwood areas are further classified into those with a complete undergrowth, those with a partial undergrowth, and those with no undergrowth. The corresponding acreages are shown in the following table.

	Complete undergrowth	Partial undergrowth	No undergrowth	Total
High-volume hardwoods	357	500	1,897	2,754
Medium-volume hardwoods	197	130	523	850
Low-volume hardwoods	39	170	646	855

Any number of acres of any crop type can receive one of two basic treatments: fell and plant conifer (treatment 1A) or fell and plant hardwood (treatment 1B). When applied to bare land, these treatments become "plant conifer" or "plant hardwood." In addition, for hardwood areas with a complete undergrowth, management has the option of felling and retaining the undergrowth (treatment 2); similarly, for hardwood areas with a partial undergrowth, management has the option of felling and enriching the undergrowth (treatment 3).

A final option is simply to postpone treatment altogether for any number of acres of any crop type.

The net discounted revenue (NDR) over the next ten years varies with treatment and crop type. These figures, in pounds per acre, are estimated in the following table:

Crop type	1A	1B	2	3	No treatment
1	287	215	228	292	204
2	207	135	148	212	148
3	157	85	98	162	112
4	487	415	—	—	371
5	337	265	—	—	264
6	87	15	—	—	61

Visual amenity requirements and a limited labor capacity dictate the following four conditions:

1. The treated area must not exceed 5,000 acres.
2. The resulting conifer area must not exceed 3,845 acres.
3. The volume of felled hardwood must not exceed 2.44 million h.ft.
4. The volume of felled conifer and mixed high forest must not exceed 4.16 million h.ft.
5. At least 500 acres must be planted with hardwood.

The conifer area in (2) is the area of newly planted conifer *plus* the untreated area of old conifer. Estimates of the average volume per acre of each of the five crops are listed in the first table.