

**MA 514 Homework #3**  
Due Monday, September 22, in Class

Graduate students should do all of these problems. Undergraduate students should do at least four but are welcome to do all five.

1. Use the result of the Matrix Tree theorem, considering the appropriate determinant, to prove that the number of labeled spanning trees is  $n^{n-2}$ .
2. Problem 2E (Graceful Labeling).
3. Problem 2F (Tree with exactly one vertex of degree  $i$  for  $2 \leq i \leq m$ ).
4. Consider the complete labeled graph  $K_n$  on  $n$  vertices. Delete any one edge. Find and justify a formula for the number of spanning trees in the resulting graph.
5. Assume that  $G$  is a connected graph. For each edge  $e$ , let  $c_e$  be the number of cycles (polygons) containing  $e$ . Prove that if  $c_e$  is odd for every edge  $e$ , then  $G$  is Eulerian.