MA 514 Homework #6
Due Monday, November 17, in Class

Graduate students should do all of these problems. Undergraduate students should do at least four but are welcome to do all five.

1. Create a two-commodity max flow problem with source vertices s_1, s_2 and respective sink vertices t_1, t_2, and integer capacities on the directed edges, with the goal of maximizing the sum of the flows $|f_1| + |f_2|$, such that this maximum sum is not an integer.

2. Problem 7B.

3. Problem 7C.

4. Let $G = (V, E)$ be a digraph with distinguished vertices $s \neq t$ and with both lower $\ell(u, v)$ and upper $c(u, v)$ bounds on its edges (u, v) such that $0 \leq \ell(u, v) \leq c(u, v)$ for every edge (u, v). Assume there is a feasible (s, t)-flow.

   (a) Prove that for any (s, t)-flow f and any (s, t)-cutset $(W, \overline{W})$ we must have $|f| \geq \ell(W, \overline{W}) - c(\overline{W}, W)$.

   (b) Starting from a feasible flow, modify the max flow algorithm appropriately to prove that the minimum value of an (s, t)-flow equals the maximum of $\ell(W, \overline{W}) - c(\overline{W}, W)$ over all (s, t)-cutsets $(W, \overline{W})$.

5. Use the previous problem to prove: Let G be a digraph with no directed polygons (i.e., an acyclic digraph). Prove that the minimum number of simple dipaths required to cover all of the edges of G is equal to the maximum number of edges, no two of which are contained in a common simple dipath in G.