

**MA515 Homework #4**  
**Due Friday, October 6**

Let us assume we have an LP in the form

$$\begin{aligned} \max z &= c^T x \\ \text{s.t. } Ax &= b \\ x &\geq O \end{aligned}$$

where the matrix  $A$  has full row rank as a result of inserting slack variables. We can represent the data in the form of a tableau  $T$ . For example, here is the tableau for the GGMC problem:

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$-z$	
1	2	1	0	0	0	120
1	1	0	1	0	0	70
2	1	0	0	1	0	100
5	4	0	0	0	1	0

Each row represents an equation. For example, the first row represents the equation  $x_1 + 2x_2 + x_3 = 120$  and the last row represents the equation  $5x_1 + 4x_2 - z = 0$  (which is equivalent to  $z = 5x_1 + 4x_2$ .) Note the identity matrix associated with the columns for the slack variables and the column  $-z$ .

Now suppose we are interested in focusing our attention on a different basis for the column space of  $A$ , say,  $B = \{1, 2, 5\}$ . We can perform row operations on the tableau  $T$  to result in a tableau  $T'$  with an identity matrix in the columns associated with the new basis (and the column labeled by  $-z$ ):

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$-z$	
1	0	-1	2	0	0	20
0	1	1	-1	0	0	50
0	0	1	-3	1	0	10
0	0	1	-6	0	1	-300

The rows of  $T'$  represent a set of four equations equivalent to the original four equations of  $T$ .

1. How can you easily read off the associated basic solution  $\bar{x}$  from  $T'$ ? Why does this work in general?
2. How can you easily read off the associated basic directions from  $T'$ ? Why does this work in general?

3. How can you easily read off the costs of the associated basic directions from  $T'$ ? Why does this work in general?
4. When contemplating a pivot, how can we determine the entering variable from  $T'$ ? Why does this work in general?
5. When contemplating a pivot, how can we determine whether the LP has unbounded objective function value from  $T'$ ? Why does this work in general?
6. When contemplating a pivot, how can we perform the ratio test using the data in  $T'$ ? Why does this work in general?
7. How can you easily read off the vector  $\bar{y}$  from  $T'$ ? Why does this work in general?