MA/STA515 Homework #7
Due Wednesday, December 1

1. Problem 6A.

2. Problem 6B.

3. Problem 6C.

4. Prove that in any sequence of $mn + 1$ distinct real numbers there must be either an increasing subsequence of length (at least) $m+1$ or a decreasing subsequence of length (at least) $n + 1$.

5. Solve one of the following problems using the Pigeonhole Principle:

   (a) Prove that for every convex three-dimensional polyhedron there are at least two faces with the same number of edges.

   (b) Prove that no matter how a set S of 10 positive integers smaller than 100 is chosen there will always be two completely different selections from S that have the same sum.

   (c) Suppose some set of 101 numbers $a_1, \ldots, a_{101}$ is chosen from the numbers $1, 2, \ldots, 200$. Prove that it is impossible to choose such a set without taking two numbers for which one divides the other evenly.

   (d) Consider a circle C with a radius of 16 and an annulus, or ring, A, with an outer radius of 3 and an inner radius of 2. Prove that wherever one might sprinkle a set S of 650 points inside C the annulus A can always be placed on the figure so that it covers at least 10 of the points.

   (e) Six circles (including their circumferences and interiors) are arranged in the plane so that no one of them contains the center of another. Prove that they cannot have a point in common.