MA614 Homework #1

Due Wednesday, January 17

1. Find the power series expansion of $\frac{1}{(1-x)^2}$ in three different ways:
   (a) Using Taylor's formula.
   (b) Squaring a known power series.
   (c) Differentiating a known power series.

2. Use Taylor's formula to show the power series expansion

$$(1+x)^\alpha = \sum_{n\geq 0} \frac{\alpha \cdot (\alpha-1) \cdots (\alpha-n+1)}{n!} \cdot x^n$$

   where α is any complex number. This is known as the Binomial Theorem.

3. Let $f(x)$ be the ogf of the sequence $(a_n)_{n\geq 0}$. Express, using $f(x)$, the ogf's of the following sequences:
   (a) $(na_n)_{n\geq 0}$.
   (b) $(n^2 a_n)_{n\geq 0}$.
   (c) $a_0, a_0 + a_1, a_0 + a_1 + a_2, \ldots$.

4. Let the sequence $(a_n)_{n\geq 0}$ be described by the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$, $n \geq 2$ and the boundary conditions $a_0 = 0$ and $a_1 = 1$.
   (a) Find the ogf.
   (b) Find an explicit formula for a_n.