MA614 Homework #10

Due Wednesday, April 11

1. Find a simple formula for the number of maximal chains in the partition lattice Π_n.

2. Prove that the rank generating function $F(P, q)$ for the power poset $C_n^{C_m}$ is the q-binomial coefficient $\begin{bmatrix} m+n+1 \\ n \end{bmatrix}$. Suggestion: Create a bijection with a certain family of lattice paths.

3. Let P be a finite join-semilattice with minimal element $\hat{0}$.
   (a) Prove that P is a lattice.
   (b) Give a counterexample to (a) when we drop the assumption that P is finite.
   (c) Give a counterexample to (a) when we drop the assumption that P has minimal element $\hat{0}$.

4. On page 102 Stanley gives the Hasse diagrams of all lattices with at most six elements. Let's refer to these lattices by the labels 1 through 25 in the obvious way—lattices 1–10 are in the first row, lattices 11–19 are in the second row, and lattices 20–25 are in the third row.
   (a) Which of these 25 lattices are (upper) semimodular? You can list your answers without justification, but be sure you know why your answers are correct. (Stanley states on page 104 that exactly eight of the lattices with exactly six elements are semimodular.)
   (b) Which of these 25 lattices are distributive? You can list your answers without justification, but be sure you know why your answers are correct.

5. Let n be an integer greater than or equal to 3. Determine the number of rank n distributive lattices such that each rank between 1 and $n-1$ (inclusive) has exactly two elements.