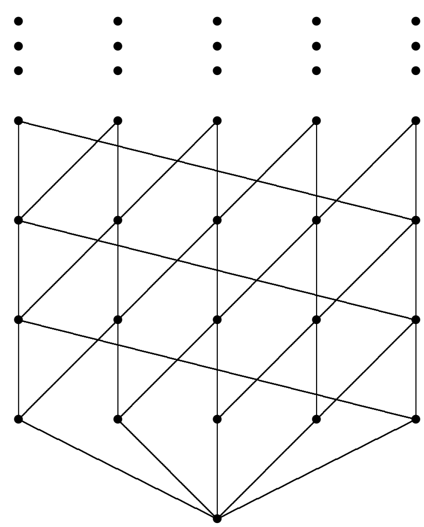
MA614 Homework #11
Due Wednesday, April 18

1. For two finite bounded posets P and Q (that is, each poset has a unique minimal element $\hat{0}$ and unique maximal element $\hat{1}$), define the diamond product of P and Q to be $P \diamond Q = [(P - \{\hat{0}\}) \times (Q - \{0\})] \cup \{\hat{0}\}$. Show that $\mu(P \diamond Q) = -\mu(P) \cdot \mu(Q)$. (Recall for a poset R with $\hat{0}$ and $\hat{1}$, $\mu(R)$ is defined to be $\mu(\hat{0}, \hat{1})$.)

2. For the Boolean algebra B_n prove:

$$\sum_{\hat{0} \le a \le b \le c \le d \le e \le \hat{1}} (\mu(\hat{0}, a))^4 \cdot (\mu(a, b))^3 \cdot (\mu(b, c))^6 \cdot \mu(c, d) \cdot (\mu(d, e))^{10} \cdot (\mu(e, \hat{1}))^2 = 2^n.$$

3. Let P_k be the poset on the set $\{\hat{0}\} \cup [\{0, 1, 2, 3, \ldots\} \times \{1, \ldots, k\}]$ for $k \geq 2$, where the cover relation is defined by the three relations (i) $\hat{0} \prec (0, j)$ for $1 \leq j \leq k$, (ii) $(n, j) \prec (n+1, j)$ for $1 \leq j \leq k$, and (iii) $(n, j) \prec (n+1, j+1)$ for $1 \leq j \leq k$, where we let $k + 1 = 1$ in the second coordinate. The Hasse diagram for a portion of the poset P_5 is drawn below:



Compute the Möbius function $\mu(\hat{0}, (n, j))$ in the poset P_k.

4. Let P be the poset on the set $\mathbf{Z}^2 = \{(a, b) : a, b \in \mathbf{Z}\}$ with the order relation

$$(a, b) \leq (c, d) \text{ if } b \leq d \text{ and } |c - a| \leq d - b.$$

Find an expression for the Möbius function $\mu((0, 0), (m, n))$ and prove it.