MA614 Homework #2

Due Wednesday, January 24

1. Let a_n be the number of compositions of n, $n \geq 0$, in which every part is odd. (Let's choose by convention to let $a_0 = 1$, since every part of the empty composition is odd.)
   (a) Find the ogf of $(a_n)_{n\geq 0}$.
   (b) Use this to prove that $a_n = F_{n-1}$, $n \geq 1$.
   (c) Give a bijection between compositions of n into odd parts, and compositions of $n-1$ into 1's and 2's, $n \geq 1$.

2. Let a_n be the number of compositions of n into parts greater than or equal to 2. (Let's choose by convention to let $a_0 = 1$, since every part of the empty composition is greater than or equal to 2.)
   (a) Find the ogf of $(a_n)_{n\geq 0}$.
   (b) Express a_n in terms of Fibonacci numbers.
   (c) Give a bijection between compositions of n into parts greater than or equal to 2, and compositions of $n-2$ into 1's and 2's, $n \geq 2$.

3. Find the generating function $\sum_{n\geq 0} a_n x^n$ where $a_n = \sum_c c_1 \cdot c_2 \cdots c_k$ as $c = (c_1, c_2, \ldots, c_k)$ ranges over all compositions of n. Suggestion: Adapt the proof of the derivation of the generating function for all compositions.

4. Find the generating function $\sum_{n\geq 0} a_n x^n$ where $a_n = \sum_c 2^{c_1-1} \cdot 2^{c_2-1} \cdots 2^{c_k-1}$ as $c = (c_1, c_2, \ldots, c_k)$ ranges over all compositions of n. Then find use this to find an explicit formula for a_n.

5. Let a_n be the number of tilings of a $3 \times 2n$ rectangle by domino tiles (recalling that a domino tile has size 1×2). Find the ogf for the sequence $(a_n)_{n\geq 0}$.

6. On your own: Try using Maple to find the first few terms of the Taylor expansion of various functions. For example, type

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taylor(1/(1-x-x^2),x=0,10);
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   to see the beginning of the ogf for the Fibonacci numbers.