

MA614 Homework #7

Due Wednesday, March 7

1. Let a_n be the number of ways of doing the following: Partition the elements of $[n]$ into nonempty blocks, then partition the blocks themselves into nonempty “superblocks.”
 - (a) Determine the exponential generating function for a_n .
 - (b) Calculate a_3 and confirm this number by explicitly enumerating all of the possibilities.
2. Find the egf and an expression for the number of permutations of $[n]$ having no 2-cycles.
3. Suppose we have a room with n children which are distinguishable. The children gather into circles by holding hands, and one child stands in the center of each circle. A circle may consist of as little as one child clasping his or her hands around the other child at the center of the circle. Let h_n be the number of ways this can be done. Find the exponential generating function for h_n .
4. Determine an expression for the number of labeled rooted trees such that each node is either a leaf (has no children) or has 3 children.
5. For n a positive integer let a_n be the number of labeled rooted trees on $[n]$ such that there is a linear order on the set of children of each node and every node has an even number of children. Determine an expression for a_n .
6. Let $f(x)$ be a formal power series that satisfies $f(0) = 0$ and the cubic equation

$$f(x)^3 - 2f(x)^2 + f(x) - x = 0.$$

Find the coefficient of x^n in $f(x)$.