# Geometry's Future: Past, Present, and Future

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# Overture: Pipe Dreams

Geometry and Music!

http://www.youtube.com/watch?v=hyCIpKAIFyo

# Act I: Geometry's Future: Past

What does my title mean??!!

# Science

What are some major advances and discoveries in science in the last 100 years that have impacted the K–12 curriculum? [Discuss!]

# Science

From "Chronology of twentieth-century science",
http://www.press.uchicago.edu/Misc/Chicago/284158.html

- 1900 Quantum theory proposed / Planck
- 1901 Discovery of human blood groups / Landsteiner
- 1905 Wave-particle duality of light / Einstein
- 1905 Special theory of relativity / Einstein
- 1906 Existence of vitamins proposed / Hopkins
- 1906 Evidence that Earth has a core / Oldham
- 1909 Idea of genetic disease introduced / Garrod
- 1909 Boundary between Earth's crust and mantle identified / Mohorovicic

# Science

- 1910 First mapping of a gene to a chromosome / Morgan and others
- 1911 Discovery of the atomic nucleus / Rutherford
- 1911 Superconductivity discovered / Onnes
- 1912 Discovery of cosmic rays / Hess
- 1912 Idea of continental drift presented / Wegener
- 1914 First steps in elucidating chemical transmission of nerve impulses: neurotransmitters / Dale; Barger; Loewi
- 1915 General theory of relativity / Einstein
- 1918 onward Synthesis of genetics with the theory of evolution by natural selection (neodarwinism) / Fisher; Haldane; Wright
- 1923 Nature of galaxies discovered / Hubble

# Science

- 1925 Description of Australopithecus africanus / Dart
- 1928 Discovery of penicillin / Fleming
- 1929 Expansion of the Universe established / Hubble
- 1929 First suggestion that Earth's magnetic field reverses / Matuyama
- 1930s Theory of chemical bonds developed / Pauling
- 1931 Birth of radioastronomy / Jansky
- 1931 First electron microscope / Ruska
- 1932 Discovery of the neutron / Chadwick
- 1935 Magnitude scale for earthquakes / Richter
- 1935 Theory of the nuclear force / Yukawa
- 1938 Nuclear reactions in stars / Bethe; von Weizsäcker
- 1938 First observation of superfluidity / Kapitza

# Science

- 1939 Discovery of nuclear fission / Meitner & Frisch
- 1943 Mutations in bacteria identified / Luria & Delbrück
- 1946 Radiocarbon dating / Libby
- 1946 Initial elucidation of the reactions involved in photosynthesis / Calvin
- 1947 Invention of the transistor / Shockley, Bardeen, and Brattain
- 1948 Big Bang theory for origin of the Universe / Gamow, Alpher, and Herman
- 1952 First polio vaccine / Salk
- 1953 Production of amino acids in “early Earth” conditions / Miller & Urey

# Science

- 1953 First determination of the amino-acid sequence of a protein / Sanger et al.
- 1953 Structure of DNA: the double helix / Watson & Crick
- 1956 Discovery of the neutrino / Cowan & Reines
- 1958 Quantum tunneling of electrons in semiconductors / Esaki
- 1958 First three-dimensional protein structure published / Kendrew et al.
- 1960 First laser / Maiman
- 1960 onward Discoveries of fossils of early Homo in East Africa / Leakeys and others

# Science

- 1961 Nature of the genetic / triplet code proposed / Crick et al.
- 1963 Discovery of quasars / Schmidt
- 1964 Existence of quarks proposed / Gell-Mann; Zweig
- 1965 Discovery of cosmic microwave background radiation / Penzias & Wilson
- 1967 First warning of an anthropogenic "greenhouse effect" / Manabe & Wetherald
- 1967 Theory of plate tectonics / McKenzie & Parker; Morgan
- 1967 Proposal that certain cell organelles are descended from free-living bacteria / Margulis
- 1968 Pulsars discovered / Hewish et al.

# Science

- 1973 Advent of genetic engineering techniques / Cohen, Boyer, and Berg
- 1973 Invention of magnetic resonance imaging / Lauterbur
- 1974 Identification of CFCs as threat to ozone layer / Molina & Rowland
- 1974 Discovery of "Lucy," Australopithecus afarensis / Johanson & Taieb
- 1977 First complete DNA sequence of an organism / Sanger et al.
- 1977 Discovery of deep-sea hydrothermal vents / Corliss et al.
- 1978 Observation of astronomical dark matter / Rubin

# Science

- 1980 First human oncogene / "cancer gene" identified / Weinberg
- 1980 Impact hypothesis for extinctions at the Cretaceous/Tertiary boundary / Alvarez et al.
- 1983 AIDS virus identified / Barré-Sinoussi et al.
- 1985 Genetic fingerprinting invented / Jeffreys
- 1985 Ozone hole discovered / Farman et al.
- 1985 Discovery of buckminsterfullerene / Kroto et al.
- 1987 Formulation of the "Out of Africa" hypothesis of human evolution using molecular data / Cann, Stoneking, and Wilson
- 1995 First extrasolar planet identified / Mayor & Queloz
- 1997 Dolly the sheep created by cloning / Wilmut et al.

# Science

- 2001 Publication of near-complete sequences of the human genome / International Human Genome Sequencing Consortium; Venter et al.

# Science

These topics were not in the immediate scientific future of a student graduating from high school in the *past*, but ARE now in the immediate scientific future of a student graduating from high school in the *present*.

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These topics were not in the immediate scientific future of a student graduating from high school in the *past*, but ARE now in the immediate scientific future of a student graduating from high school in the *present*.

As a result, some changes have been made in what is taught in school.

# Geometry

Now let's think about geometry. First, consider the geometry standards:

# Geometry Standards

Geometry, — Plane and solid geometry, including problems in mensuration of plane and solid figures, and original propositions in plane geometry.

# Geometry Standards

Geometric education should begin in the kindergarten or primary school, where the child should acquire familiarity through the senses with simple geometric forms, by inspecting, drawing, modelling, and measuring them, and noting their more obvious relations. This study should be followed, in the grammar school, by systematic instruction in concrete (or observational) geometry, of which geometric drawing should form a part. Such instruction should include the main facts of plane and solid geometry, treated as matters of observation, and not as exercises in logical deduction, without however necessarily excluding the beginnings of deductive proof as soon as the pupil is ready for them.

# Geometry Standards

Concrete geometry is believed to have important educational value, and to prepare an excellent foundation for the later study of formal geometry. It belongs, however, to the earlier stages of school work, and should not be postponed until the time that belongs to direct preparation for college or the scientific school.

# Geometry Standards

In teaching formal geometry, stress should be laid from the outset on accuracy of statement and elegance of form, as well as on clear and strict reasoning. As soon as the pupil has begun to acquire the art of rigorous demonstration, his work should cease to be merely receptive, he should be trained to devise constructions and demonstrations for himself, and this training should be carried through the whole of the work in plane geometry. Teachers are advised, in their selection of a text-book, to choose one having a clear tendency to call out the pupil's own powers of thought, prevent the formation of mechanical habits of study, and encourage the concentration of mind which it is a part of the discipline of mathematical study to foster.

# Geometry Standards

The subject of geometry, not a particular treatise, is what the pupil should be set to learn; and its simpler methods and conceptions should be made a part of his habitual and instinctive thought. Lastly, the pupil should be stimulated to good work by interest in the study felt and exhibited by the teacher.

# Geometry Standards

The requirement in geometry embraces the following topics: the general properties of plane rectilinear figures; the circle and the measure of angles; similar polygons; areas; regular polygons, and the measure of the circle; the relations of planes and lines in space; the properties and measure of prisms, pyramids, cylinders, and cones; the sphere and the spherical triangle.

# Geometry Standards

The time which it is recommended to assign to the systematic study of the requirement in formal geometry is the equivalent of a course of five lessons a week for one school year; but it is believed to be advisable to extend this allowance of time over two years.

# Geometry Standards

Where does this come from? [Guess!]

# Geometry Standards

Where does this come from? [Guess!]

*The Harvard University Catalog*, 1898–99, geometry entrance requirements for admission examination.

# Geometry's Future: Past

What geometry lay in the *future* of a student graduating from high school in the *past*?

# Geometry's Future: Past

Some 1898 Harvard undergraduate courses involving geometry:

- Plane Analytic Geometry
- Plane and Solid Analytic Geometry
- Solid Geometry
- Trigonometry and Plane Analytic Geometry
- Differential and Integral Calculus
- Modern Methods in Geometry—Determinants
- Quaternions with Applications to Geometry and Mechanics

# Geometry's Future: Past

- Astronomy—Practical Astronomy
- Astronomy—Spherical Astronomy
- Engineering—Descriptive Geometry
- Engineering—Stereotomy, Shades, Shadows, and Perspective
- Engineering—Surveying

# Interlude: Andrew Hamilton MacPhail

Speaking of admissions tests. . .

# Andrew Hamilton MacPhail

Brown University Professor of Educational Psychology, served on the College Entrance Commission which developed the SAT. Goal: Move away from different entrance exams at each Ivy League college and open the doors to students demonstrating aptitude regardless of the pre-college institution. My grandfather.
From Martha Mitchell's *Encyclopedia Brunoniana*.

# Act II: Geometry's Future: Present

What are some major advances and discoveries in geometry in the last 100 years that have impacted what lies ahead in the geometric future of present-day graduating high school students? And what can we do to prepare them? What elements might be incorporated into the K–12 curriculum? [Discuss!]

# Geometry's Future: Present

Acknowledgment: Idea for talk title comes from *Geometry's Future* by COMAP, edited by Joe Malkevitch. See also his website: Geometry in Utopia II, `http://www.york.cuny.edu/~malk/utopia.html`.

Thanks also to

- David Royster, "Geometry in Society", University of Kentucky
- Nathalie Sinclair, *The History of the Geometry Curriculum in the United States*, IAP, 2008.

# Geometry's Future: Present

From Malkevitch: "In recent years, there has been a tremendous surge in research in geometry. This surge has been the consequence of the development of new methods, the refinement of old ones, and the stimulation of new ideas both from within mathematics and from other disciplines, including Computer Science. Yet during this period of growth, education in geometry has remained stagnant. Not only are few of the new ideas in geometry being taught, but also fewer students are studying geometry."

# Geometry's Future: Present

A selection of thoughts, but by no means comprehensive. . .

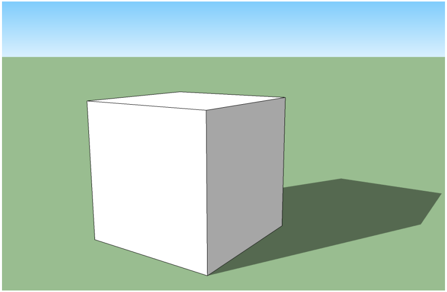
# Discrete Geometry

Making arrangements and counting various collections and arrangements of geometric objects.

Applications: Geometric modeling, robotics, computer graphics, crystallography,. . .

# Discrete Geometry

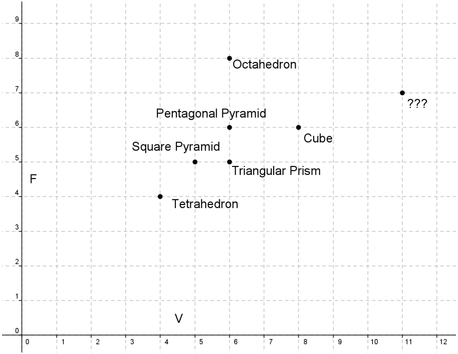
Example: A cube, with $(V, E, F) = (8, 12, 6)$, satisfies Euler's relation: $V + F = E + 2$, which holds for all convex polyhedra.



A commonly discussed topic in the current K–12 curriculum.

# Discrete Geometry

- Is there a polyhedron with $(V, E, F) = (11, 16, 7)$?
- For what triples of numbers $(V, E, F)$ do polyhedra exist?



# Discrete Geometry

Thomas Hales solves Kepler Conjecture after 400 years.

# Computational Geometry

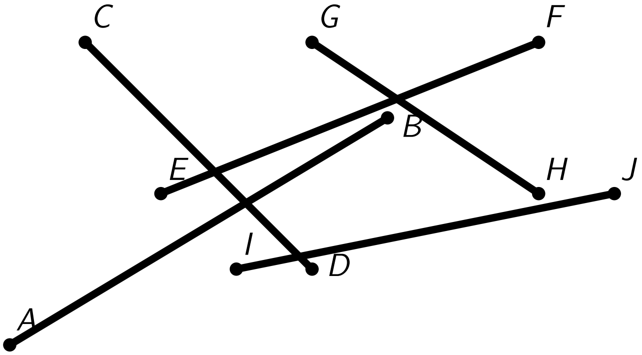
Geometry has algorithms!

Applications: Numerical analysis, robotics, computer-aided design and engineering, computer graphics, geographic information systems (e.g., GPS), route planning, integrated circuit design, computational chemistry and biology, video game design, geology (e.g., earthquake location),...

Also, the mathematics behind all the interactive geometry software!

# Computational Geometry

Example: Which of the following pairs of line segments intersect?



# Computational Geometry

Which of the following pairs of line segments intersect?

- $(-1, 0)$, $(4, 3)$
- $(0, 4)$, $(3, 1)$
- $(1, 2)$, $(6, 4)$
- $(3, 4)$, $(6, 2)$
- $(2, 1)$, $(7, 2)$

What algorithm can be used to efficiently solve such problems?

# Differential Geometry

The geometry of smooth surfaces and objects; bringing calculus and geometry together.

Foundational for general relativity and the geometry of the universe, for example.

# Geometric Visualization and Modeling

Especially driven by the development of computer graphics.

Applications: CAD, robotics, computer graphics, computer vision, animated movies, computer gaming, medical imaging, astronomy, art,. . .

Example: The Visible Human Project of the U.S. National Library of Medicine.
Website: http://www.nlm.nih.gov/research/visible/visible_gallery.html
Video: http://www.youtube.com/watch?v=iWP2HnPSMyo.

# Geometric Visualization and Modeling

Examples:

- Geometer's Sketchpad, http://www.dynamicgeometry.com
- Cabri, http://www.cabri.com
- WinGeom, free, http://math.exeter.edu/rparris/wingeom.html
- GeoGebra, free, http://www.geogebra.org

# Geometric Visualization and Modeling

- SketchUp, free, `http://sketchup.google.com`. Ask your students to watch the videos and teach you. [Brief Demonstration]
- POV-Ray, free sophisticated ray-tracer for amazing images, `http://www.povray.org`
- Blender, free, `http://www.blender.org`. You can even make video games with this.

# Geometric Visualization and Modeling

Jessie Clark Middle School Geometry Project, with teacher Dr. Craig Schroeder.

http://www.ms.uky.edu/~lee/jessieclark/jessieclark.html.

One student's reflection: "One skill used was angle measures. When you had to rotate things, you had to know the angle measurement to get the object in the correct position. Another skill was scaling an object. You had to know proportions to scale the object correctly. One last skill you had to know was lines. You had to know things about lines in order to actually construct objects."

# Central Role of Transformations and Symmetry

1872 Klein's Erlanger Programm — classify geometries on the basis of transformations. Led to a deeper framework for understanding non-Euclidean geometries.

Transformations and Symmetry—deep mathematical themes underlying many areas, including algebra (e.g., analytical geometry), biology (e.g., chirality), chemistry (e.g., crystallography, molecular symmetry), physics (e.g., classification of particles, relativity), computer graphics,. . .

# Transformations

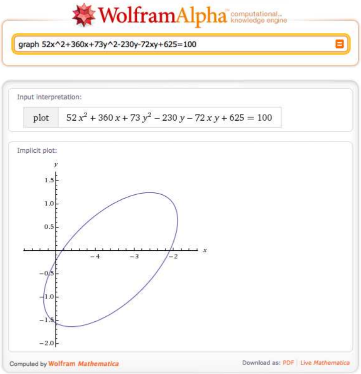
Example: A problem from my 1970–71 high school analytical geometry course (Baltimore County public school system):

Apply the appropriate transformations to identify and graph the conic satisfying the equation

$$52x^2 + 360x + 73y^2 - 230y - 72xy + 625 = 100.$$

# Transformations

WolframAlpha, `http://www.wolframalpha.com`, is a powerful geometric as well as algebraic tool.



# $n$-Dimensional Geometry

Regard ordered tuples of $n$ variables as points in $n$-dimensional space, and analyze as geometric objects.

Applications in operations research, physics, computer science, astronomy, cosmology,. . .

Example: A hypercube is just the set of all points $(w, x, y, z)$ for which each coordinate lies between 0 and 1. Visualizing it is a different matter!

Famous Banchoff video: “The Hypercube: Projections and Slicing,” http://www.math.brown.edu/~banchoff/Hypercube.html.

# $n$-Dimensional Geometry

Example: A typical linear programming problem from high school algebra.

The Dirt Bike Dilemma from NCTM's Illuminations, `http://illuminations.nctm.org`.

This is a two dimensional problem—How many Riders and how many Rovers should be assembled? Solved by graphing linear inequalities.

# $n$-Dimensional Geometry

Suggestion: Push this into three dimensions, since that's the world we live in.
Example: Find a point $(x, y, z)$ satisfying

$$\begin{aligned} x + y &\leq 12 \\ x + z &\leq 12 \\ y + z &\leq 12 \\ x &\geq 0 \\ y &\geq 0 \\ z &\geq 0 \end{aligned}$$

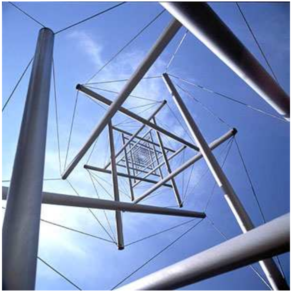
- that has the largest value of $3x + 4y + 5z$.
- that has the largest value of $2x + 3y + 6z$.

# $n$-Dimensional Geometry

Next to statistics and simulation, linear programming is one of the most widely used tools in operations research and industrial engineering.

# Geometry in Art

A casual search will reveal thousands of examples of elements of geometry in art.
Example: Tensegrity sculptures.



http://enpointepilates.com/homebox-3/tensegrity-tower-vertical

# Geometry in Art

Engage students in art projects directly related to a geometric strand. Collaborate with the art department.

# Interlude: A Personal Journey

Reflecting on my own school age experience. Two influences proceeding in parallel:

# Formal Geometry

Formal High School courses, Baltimore County Public School system.

- Algebra I — Grade 8
- Algebra II — Grade 9
- Geometry (SMSG, axiomatic) — Grade 10
- Trigonometry and Analytic Geometry — Grade 11, full year course. Included an introduction to the algebra and geometry of complex numbers, vectors, matrices, applications of translations and rotations to conics.
- College Algebra — Grade 11, full year course. Included proofs by induction, analyzing and graphing functions of two variables, introduction to groups.
- Calculus — Grade 12.

# Informal Geometry

Grabbing books off the family bookshelf and the school library:

- Collections of *Scientific American* articles by Martin Gardner
- Cundy and Rollett, *Mathematical Models*
- Steinhaus, *Mathematical Snapshots*
- Holden, *Shapes, Space, and Symmetry*
- Physical math puzzles
- Origami, especially unit origami

# Reflection

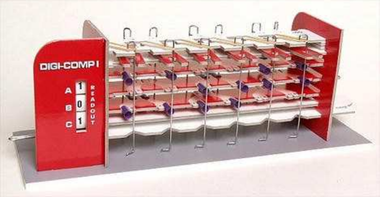
I was personally profoundly impacted by a rich mathematical environment for formal work and informal play. How can we enhance the environment for our students?

Also, do these formal high school courses still exist? Should they? Can they?

# Act III: Geometry's Future: Future

What lies in the geometric future of our future students?

# Adaptability




http://retrothing.typepad.com/photos/uncategorized/digicomp.jpg

My first computer in 1965 was the 3-bit Digicomp I. In high school I wrote programs to graph geometrical objects by printing labeled dots on sheets of paper that I had to connect by hand with a pencil. Now I have access to unimaginably more powerful machines.

# Adaptability

It is hard to accurately predict what the next 30 or 50 or 100 years will bring, so we must learn to be flexible to prepare adaptable teachers and students.

# New Applications

New applications of geometry appear every day. We must keep our antennae up for classroom connections. Our students may be more aware of these connections than we are, and wonder what the relationship is to school geometry.

Example: Radio Lab podcast "Lost," starting at the 10 minute mark. `http://www.radiolab.org`.

# Technology

Advances in technology — what will become available as it becomes cheaper, and what will be developed?

Example: Three-D printers.

http://video.nytimes.com/video/2010/09/13/technology/1248068999175/desktop-manufacturing.html

# Technology

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What else is to become readily accessible in the classroom of the future? Holograms? Virtual reality systems?

# Power of Modern Media

Our students are immersed in media that did not exist 100 years ago. Geometry is seen everywhere, yet is poorly connected with their school experiences. How can we take advantage of such media and strengthen connections? This includes students creating mathematical media of their own.

Example: Vi Hart — Infinity Elephants.
`http://www.youtube.com/watch?v=DK5Z709J2eo`.

# What is Your ZPD?

Zone of Proximal Development: "The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers," L.S. Vygotsky: *Mind in Society: Development of Higher Psychological Processes*.

# What is Your ZPD?

We have lots of problems to solve! Let's (probably inappropriately) expand the application of this concept. When it comes to developmental change, what is the ZPD of

- Your district?
- Your school, college, or university?
- Your department?
- Your own teaching?

What are the implications for professional development? For policy?

# What is Your ZPD?

Change is unlikely to happen if what is proposed is what you are already doing.

Change is unlikely to happen if what is proposed is too far away or too overwhelming from what you are already doing.

Choose measured, thoughtful, creative, manageable steps forward. . . .

# The Geometric Forest

Balance detail with perspective as you guide your students through their geometrical forest, which extends well beyond your classroom.

# Finale

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Google, of course!

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Google, of course!

Here is the result of searching for "geometry" within Google Images—a "small" ($< 1000$) selection of the results (with a few extras thrown in for personal taste). [Video made with iMovie]

Remember: Your students are already adept at making videos. Challenge them to try their hand with mathematics topics!