Syllabus for A&S 100 Section 004
Introduction to Contemporary Mathematics
Fall 2002

Basic Skills Requirement: This course will fulfill the University Studies Basic Skills requirement, and can be followed by PHI 120 and STA 200 to fulfill the Inference requirement. Note, however, that A&S 100 WILL NOT COVER THE PREREQUISITE MATERIAL FOR CALCULUS COURSES.

Course: A&S 100 Section 004, MWF 1–1:50, CP 220

Instructor: Carl Lee

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Office Hours: MWF 2–2:50, and by appointment.

Prerequisites: Two years of high school algebra and a Math ACTE score of 19 or above, or MA 108R, or math placement test.

Text: *Mathematics Beyond the Numbers*, George T. Gilbert and Rhonda L. Hatcher, Wiley, 2000.

Material to be Covered: We will be covering sections 1.1-1.5, 2.1-2.4, 8.1-8.4, 7.1-7.3, 9.1, and 3.1-3.5 of your text. Topic include, but are not limited to Voting Methods, Apportionment, Modular Arithmetic and Check Digits, Polyhedra and Tilings, Alternate Move Games, and the Mathematics of Money. At times during the semester, your instructor may provide you with supplemental material.

Homework and Quizzes: Homework will be assigned on a regular schedule. Late homework will not be accepted (unless it is the result of an officially excused absence). There will be occasional, possibly unannounced, quizzes, during the semester. Missed quizzes may not be made up (unless this is the result of an officially excused absence).

Certain homework assignments will be designated "Writing Assignments." These assignments must be **typed.** Any Writing Assignment which is not typed will not be accepted. Writing Assignments will be graded on grammar, organization, and content.

Be aware that not all assigned homework problems will be graded, but answer keys will be available in the math library after the assignment is due. Since you are responsible for all of the assigned problems, not just those which are graded, you should use these keys to check all of your answers. Occasionally, answer keys do contain errors so please ask your instructor about any answer you do not understand.

Finally, most homework assignments include reading assignments. You are expected to read each section **before** coming to class. You are responsible for **all** of the material in the reading whether or not it is discussed in class.

Exams: There will be three in-class exams and a final. Tentatively, the first three exams are scheduled for Wednesday 25 September 2002, Wednesday 23 October 2002, and Wednesday 20 November 2002. Please see the Fall 2002 Course Schedule Book to determine the day and time of your final exam.

Grading:

Homework and Quizzes	150
Three Exams	300
Final Exam	150
Total	600 points

A	540–600 points
B	480–539
C	420–479
D	360–419
E	0–359

The University of Kentucky *Bulletin* explains: Grade A represents exceptionally high achievement as a result of aptitude, effort, and intellectual initiative. Grade B represents a high achievement as a result of ability and effort. Grade C represents average achievement. Grade D represents the minimum passing grade. Grade E represents unsatisfactory performance and indicates failure in the course.

University Studies Program: A&S 100 is part of the University Studies Program. As such, it has a writing component. Students will be expected to write coherently on assignments and exams.

Working Together: It is ok to work together on homework. However, when it comes time for you to write up the solutions, I expect you to do this completely on your own, and it would be best for your own understanding if you put aside your notes from the discussions with your classmates and wrote up the solutions entirely from scratch. Working together on exams, of course, is expressly forbidden.

Calculator: Students may use a graphing calculator on exams and homework, but the instructor reserves the right to clear the memory of any calculator during an exam. Any attempt to store notes in the memory of the calculator for an exam will be considered cheating.

Absences: You are expected to attend every class. If you are not able to turn in a homework assignment, take a quiz, or take an exam because of an unexcused absence, you will not be able to turn the homework in late or take the quiz or exam. A University excuse from a scheduled class activity such as an exam must be presented in writing no later than two weeks prior to the date of the absence. An absence due to illness or family emergency may be excused, provided that you can supply acceptable written evidence if required, and that you notify the lecturer or the teaching assistant *as soon as possible*. Notification is almost always possible immediately upon occurrence of an emergency. If you're too sick to telephone, you can get a friend to do it. Failure to make such timely notification may result in denial of your request. For an explanation of valid excused absences, refer to U.K.'s *Student Rights and Responsibilities*, http://www.uky.edu/StudentAffairs/Code/part2.html.

Cheating: The University's *minimum penalty* for cheating or plagiarism is *a failure in the course*. Cheating or plagiarism can lead to expulsion from the university. See *Student Rights and Responsibilities*, http://www.uky.edu/StudentAffairs/Code/part2.html, for information on cheating, plagiarism, and penalties. It's not worth it, so don't do it.

Expectations: I expect that everyone will maintain a classroom conducive to learning. I like an informal atmosphere, but it must be orderly. Thus, everyone is expected to behave with basic politeness, civility, and respect for others. In particular, talking in class is ok if it's part of a class discussion or with me. Private communications are not, especially during quizzes and tests. Neither are reading extraneous materials, using electronic equipment, or sleeping.

Suggestions: Suggestions for improvement are welcome at any time. Any concern about the course should be brought first to my attention. Further recourse is available through the offices of the Department Ombud and the Department Chair, both accessible from the Main Office in 715 Patterson Office Tower.

Text *Mathematics Beyond the Numbers* by George T. Gilbert and Rhonda L. Hatcher, ISBN 0-471-13934-3.

Calendar The calendar below gives the dates of exams and other important dates for the course.

Wed, 28 Aug	§1.1 *Plurality and Runoff Methods*
Fri, 30 Aug	§1.2 *Borda's Method*
	Assignments #0 and #1 due
Mon, 02 Sept	Labor Day – No Classes
Wed, 04 Sept	§1.3 *Head-to-Head Comparisons*
	Assignment #2 due
Fri, 06 Sept	Worksheet Day
	Assignment #3 due
Mon, 09 Sept	§1.4 *Head-to-Head Comparisons*
Wed, 11 Sept	§1.5 *The Search for an Ideal Voting System*
	Assignment #4 due
Fri, 13 Sept	Worksheet Day
	Assignment #5 due
Mon, 16 Sept	§2.1 *Quota Methods*
Wed, 18 Sept	§2.2 *Divisor Methods*
	Assignment #6 due
	Last day to withdraw from a class without receiving a W.
Fri, 20 Sept	Worksheet Day
	Assignment #7 due
Mon, 23 Sept	Review
Wed, 25 Sept	Exam 1
Fri, 27 Sept	§2.3 *The Huntington-Hill Method*
	Assignment #8 due
Mon, 30 Sept	§2.4 *The Search for an Ideal Apportionment Method*
	Assignment #9 due
Wed, 02 Oct	Worksheet Day
	Assignment #10 due
Fri, 04 Oct	Fall Break – No Classes
Mon, 07 Oct	§8.1 *Divisibility and Primes*
Wed, 09 Oct	§8.2 *Modular Arithmetic*
	Assignment #11 due
Fri, 11 Oct	Worksheet Day
	Assignment #12 due

Mon, 14 Oct	§8.3 *Divisibility Tests*
Wed, 16 Oct	§8.2 *Check Digits*
	Assignment #13 due
Fri, 18 Oct	Worksheet Day
	Assignment #14 due
Mon, 21 Oct	Review
Wed, 23 Oct	Exam 2
Fri, 25 Oct	§7.1*Polygons*
	Assignment #15 due
	*Last day to withdraw from a class.
Mon, 28 Oct	§7.2 *Tilings*
	Assignment #16 due
Wed, 30 Oct	§7.3 *Polyhedra*
	Assignment #17 due
Fri, 01 Nov	Worksheet Day
	Assignment #18 due
Mon, 04 Nov	More on Polyhedra
Wed, 06 Nov	§9.1 *Alternate Move Games*
Fri, 08 Nov	Worksheet Day
	Assignment #19 due
Mon, 11 Nov	§9.1 *Alternate Move Games*
Wed, 13 Nov	§9.1 *Alternate Move Games*
	Assignment #20 due
Fri, 15 Nov	Worksheet Day
	Assignment #21 due
Mon, 18 Nov	Review
Wed, 20 Nov	Exam3
Fri, 22 Nov	§3.1 Powers, Roots, and Logarithms
	Assignment #22 due
Mon, 25 Nov	§3.2 *Simple Interest*
	Assignment #23 due
Wed, 27 Nov	Worksheet Day
	Assignment #24 due
T&F, 28-9 Nov	Thanksgiving Holiday – No Classes
Mon, 02 Dec	§3.3 *Compound Interest*
Wed, 04 Dec	§3.4 *The Rewards of Systematic Savings*
	Assignment #25 due
Fri, 06 Dec	Worksheet Day
	Assignment #26 due
Mon, 09 Dec	§3.3 *Amortized Loans*
Wed, 11 Dec	Worksheet Day
	Assignment #27 due
Fri, 13 Dec	Review
	Last Day of Classes
M-F, 16-20 Dec	Finals Week

Name:__________________________

Test 1
Voting Methods and Apportionment
AS 100
Fall 2002

Objectives.
You should be able to do the following:

1. Determine the winner of a plurality election.
2. Determine the winner of a plurality election with runoff(s).
3. In a plurality election, determine the minimum number of votes a candidate needs to win.
4. In a plurality election in which part of the votes have been counted, determine the minimum number of the remaining votes a candidate needs to win.
5. Recognize that the media often "calls" an election before it is mathematically sound to do so. State an example from American history in which the winner of an election differed from the winner projected by the media.
6. Provide an example in which the winner from the plurality method is different than the winner from the plurality method with a run-off.
7. Discuss the effects of strategic voting on the plurality method.
8. Determine the winner of an election using Borda's method.
9. Discuss the effects of strategic voting on elections determined by Borda's method.
10. Show that $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.
11. Determine a formula for the sum of all Borda counts in an election with v voters and c candidates.
12. Determine the Condorcet winner of an election, if one exists.
13. Provide an example of an election in which there is no Condorcet winner.
14. Recognize the role of single peaked preference rankings in determining a Condorcet winner.
15. Determine the winner of an election using the approval voting method.
16. Compare and contrast the plurality method, the plurality method with runoff(s), Borda's method, the head-to-head comparison method, and the approval voting method in determining the winner of an election. List pros and cons of using these five different methods to determine a winner of an election.

17. Define universal domain, Pareto optimality, nondictatorship, and independence from irrelevant alternatives.

18. State Arrow's Impossibility Theorem.

19. Define anonymity and neutrality.

20. Discuss

    (a) The Plurality Method,

    (b) The Plurality Method with Runnoff(s),

    (c) Borda's Method,

    (d) Head-to-Head comparison Methods, and

    (e) The Approval Voting Method

    with respect to universal domain, Pareto optimality, non-dictatorship, independence from irrelevant alternatives, anonymity, and neutrality.

21. Apportion the seats of a house using:

    (a) Hamilton's Method

    (b) Lowndes' Method

    (c) Jefferson's Method

    (d) Webster's Method

22. Recognize that Hamilton's method and Lowndes' method are quota methods.

23. Recognize that Jefferson's method and Webster's method are divisor methods.

24. Find threshold divisors for Jefferson's and Webster's methods. Use these threshold divisors to find an appropriate divisor for these methods.

25. Compare and contrast Hamilton's, Lowndes', Jefferson's, and Webster's, methods of apportionment. Which methods favor smaller states? Which methods favor larger states?

Name:__________________________

Test 2
Apportionments and Number Theory
AS 100
Fall 2002

Objectives.
You should be able to do the following:

1. Apportion the seats of a house using the Hill-Huntington method.
2. Find threshold divisors the Hill- Huntington methods. Use these threshold divisors to find an appropriate divisor for the Hill-Huntington method.
3. Compare and contrast Hamilton's, Lowndes', Jefferson's, Webster's, and the Hill-Huntington Methods of apportionment. Which methods favor smaller states? Which methods favor larger states?
4. Define the Quota Property, the House Size Property, and The Population Property.
5. Identify the Alabama Paradox and note that it arose from and application of Hamilton's method.
6. Recognize that Jefferson's method is expected to violate the Quota Property more frequently than the Hill-Huntington Method which is expected to violate the Quota Property more frequently than Webster's Method.
7. State why the House Size Property might be irrelevant in determining an apportionment method for the U.S. House of Representatives.
8. State:
   (a) No divisor method satisfies the Quota Property.
   (b) Every divisor method satisfies the House Size Property.
   (c) An apportionment method satisfies the Population Property if and only if it is a type of divisor method in which the rounding depends on the number of states and the house size.
   (d) No apportionment method satisfies both the Quota Property and the Population Property.
9. Recognize integers.
10. For integers a and $b \neq 0$ define:
    (a) b **divides** a, $b|a$

(b) a **is divisible by** b

11. For integers a and b define:

    (a) b **divides** a, $b|a$

    (b) a **is divisible by** b

12. Recognize and use the notation $b|a$ for b divides a.

13. Define **factor, divisor, prime number,** and **composite number.**

14. State the Fundamental Theorem of Arithmetic.

15. Know that there are an infinite number of prime numbers.

16. Be able to argue that there are an infinite number of prime numbers.

17. Apply the Test for Primality to determine if a number is prime or composite.

18. Prove the Test for Primality.

19. State the Division Algorithm.

20. Given integers a and $b \neq 0$, use the Division Algorithm to find the quotient and the remainder when a is divided by b.

21. Given integers a and b, find $\gcd(a, b)$ using:

    (a) the prime factorizations of a and b.

    (b) the Euclidean Algorithm.

22. For integers $n > 0$, a, and b define:

    (a) a mod n

    (b) **congruence modulo** n, $a \equiv b(modn)$

23. For integers a, b and n with $n > 0$:

    (a) Find $a$mod$n$.

    (b) Determine if $a \equiv b$ (mod n)

24. Know and be able to apply the properties of modular arithmetic on page 483 of your text.

25. Know and be able to apply the divisibility tests for $2, 3, 4, 5, 6, 7, 8, 9, 10,$ and 11.

26. Prove the divisibility tests for $2 - 6$ and $8 - 11$.

27. Derive and prove a divisibility test for 2^k.

28. Calculate check digits according to various check digit formulas.

29. Use check digits to detect errors in data.

30. Recognize that the check digit formulas discussed in class do not detect errors with 100% accuracy.

Name:___________________________

Test 3
Tilings, Polyhedra, and Game Theory
AS 100
Fall 2002

Objectives.

You should be able to do the following:

1. Demonstrate understanding of the following terms:
    - polygon
    - $n-$gon
    - side
    - vertex (vertices) of a polygon
    - convex polygon
    - concave polygon
    - (interior) angles of a polygon
    - regular polygon
    - equiangular polygon
    - equilateral polygon
    - tiling
    - tessellation
    - monohedral tiling
    - edge-to-edge tiling
    - vertex of a tiling
    - regular tiling
    - vertex type of a vertex in a tiling
    - semiregular tiling
    - periodic tiling
    - aperiodic tiling
    - polyhedron; polyhedra
    - face of a polyhedron
    - edge of a polyhedron
    - vertex of a polyhedron

- convex polyhedron
- concave polyhedron
- Euler's Formula for Convex Polyhedron
- regular polyhedron; Platonic solids
- semiregular polyhedron
- vertex type of a polyhedron
- prism
- antiprism
- Archimedean solids
- alternative move games
- move
- game tree
- partial game tree
- strategy
- optimal strategy
- compressed game tree
- partial compressed game tree

2. Understand the relationship between convexity and concavity and the interior angles of a polygon.

3. Recognize that the common name for a regular triangle is "equilateral triangle."

4. Recognize that the common name for a regular quadrilateral is "square."

5. For $n \geq 4$, know that there are equilateral $n-$gons which are not regular. Provide an example of such an $n-$gon.

6. For $n \geq 4$, know that there are equiangular $n-$gons which are not regular. Provide an example of such an $n-$gon.

7. Know that the sum of the interior angles of a triangle is 180^o and use this fact to argue that the sum of the interior angles of a convex $n-$gon is $(n-2)180^o$.

8. For a given $n-$gon, use triangles to show that the sum of the interior angles is $(n-2)180^o$.

9. Know that the sum of the interior angles of any $n-$gon is $(n-2)180^o$.

10. Use the formula for the sum of the interior angles of an $n-$gon to derive the formula for the measure of an interior angle of a regular $n-$gon.

11. Know that the only regular tilings use equilateral triangles, squares, or regular hexagons.

12. Prove that the only regular tilings use equilateral triangles, squares, or regular hexagons.
13. Know that there are only 8 semiregular tilings.
14. Provide an example of regular polygons which fit around a vertex but do not yield a semiregular tiling of the plane.
15. Know that the order of regular polygons about a vertex can affect whether or not a vertex type produces a semiregular tiling of the plane.
16. Given any triangle or quadrilateral, demonstrate an edge-to-edge, monohedral tiling of the plane using only the triangle or quadrilateral.
17. Know that only certain types of pentagons and hexagons can be used to create edge-to-edge, monohedral tilings of the plane.
18. Know that it is not possible to form monohedral tilings of the plane with a *convex* $n-$gon for $n \geq 7$.
19. Know that some *concave* $n-$gons with 7 or more sides can give rise to tilings of the plane.
20. Use the methods on pp. 437-439 of your text to create new shapes from rectangles and triangles that will tile the plane.
21. State Euler's Formula for Convex Polyhedra.
22. Apply Euler's Formula to find an Unknown Polyhedron.
23. Know that there are only 5 regular polyhedra: the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron.
24. Analyze tic-tac-toe games, Nim games, and other alternate move games using game trees, partial game trees, and oral arguments.
25. Analyze games of chance using game trees, partial game trees, and oral arguments.

Name:____________________________

Test 4
The Mathematics of Money
AS 100
Fall 2002

Objectives.
You should be able to do the following:

1. Demonstrate understanding of the following terms:
   (a) simple interest
   (b) principal
   (c) present value
   (d) future value
   (e) compound interest
   (f) annual percentage yield; APY
   (g) inflation
   (h) systematic savings plan
   (i) amortized loan; amortization schedule
   (j) loan balance
2. Know and be able to apply the following rules of exponents:
   (a) For m and n positive integers and $x \neq 0$,
      - $x^n = x \cdot x \cdot x \cdot \cdots \cdot x$ where there are n x's
      - $(x^{-n}) = \frac{1}{x^n}$

   (b) For a positive number x and any exponents m and n,
   $$(x^m)^n = x^{mn}$$
   (c) For a positive number x and n a positive integer,
   $$\sqrt[n]{x} = x^{\frac{1}{n}}$$
   (d) For $x > 0$ and $m \neq 0$,
   $$(x^m)^{\frac{1}{m}} = x^1 = x$$
3. Know and be able to apply the following rule of logarithms:

(a) For any positive number x and any real number m,

$$\log x^m = m \log x$$

4. Solve equations involving exponents and logarithms.

5. Recognize that rounding answers at intermediate steps of interest calculations can produce significant errors in the final answer.

6. Use the following formulas at appropriate times to do financial calculations:

   (a) $I = Prt$

   (b) $F = P(1 + rt)$

   (c) $F = P\left(1 + \frac{r}{n}\right)^{nt}$

   (d) $APY = \left(1 + \frac{r}{n}\right)^n - 1)$

   (e) $F = D\left(\frac{(1+\frac{r}{n})^{nt}-1}{\frac{r}{n}}\right)$

   (f) $P = R\left(\frac{1-(1+\frac{r}{n})^{-nt}}{\frac{r}{n}}\right)$

7. Calculate a loan balance.

8. Construct an amortization table.

* Remember that Test 4 is cumulative. Therefore, you are also responsible for all material covered on previous exams.