REGULAR TILINGS

Description: A regular tiling is a tiling of the plane consisting of multiple copies of a single regular polygon, meeting edge to edge. How many can you construct?

Comments: While these tilings can be easily found by trial and error, and all are quite familiar, they do give some insight into the geometry of regular polygons.

Hints: There are only three regular tilings.

References: Kappraff, *Connections: The Geometric Bridge Between Art and Science*, McGraw-Hill, 1991, Chapter 5.

REGULAR TILINGS

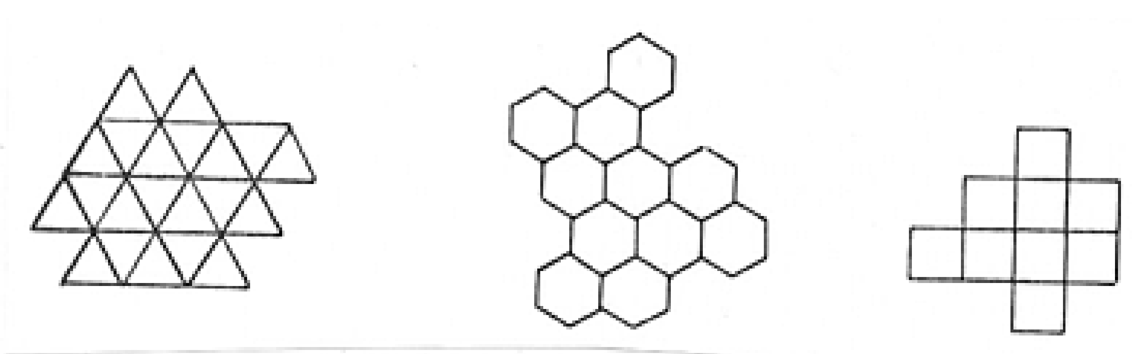

Illustration: Kappraff, p. 179

SEMIREGULAR TILINGS

Description: A semiregular tiling is a tiling of the plane consisting of at least two different types of regular polygons. In addition, the pattern of polygons meeting at a common juncture point (or vertex) must be the same for each point. How many can you construct?

Comments: Some of these tilings are not immediately obvious. You can calculate or eliminate some possibilities by making a table of the measures of the interior angles of regular polygons, since the sum of these angles for the polygons meeting at a common vertex must be 360 degrees.

Hints: There are only eight semiregular tilings.

References: Kappraff, *Connections: The Geometric Bridge Between Art and Science*, McGraw-Hill, 1991, Chapter 5.

SEMIREGULAR TILINGS

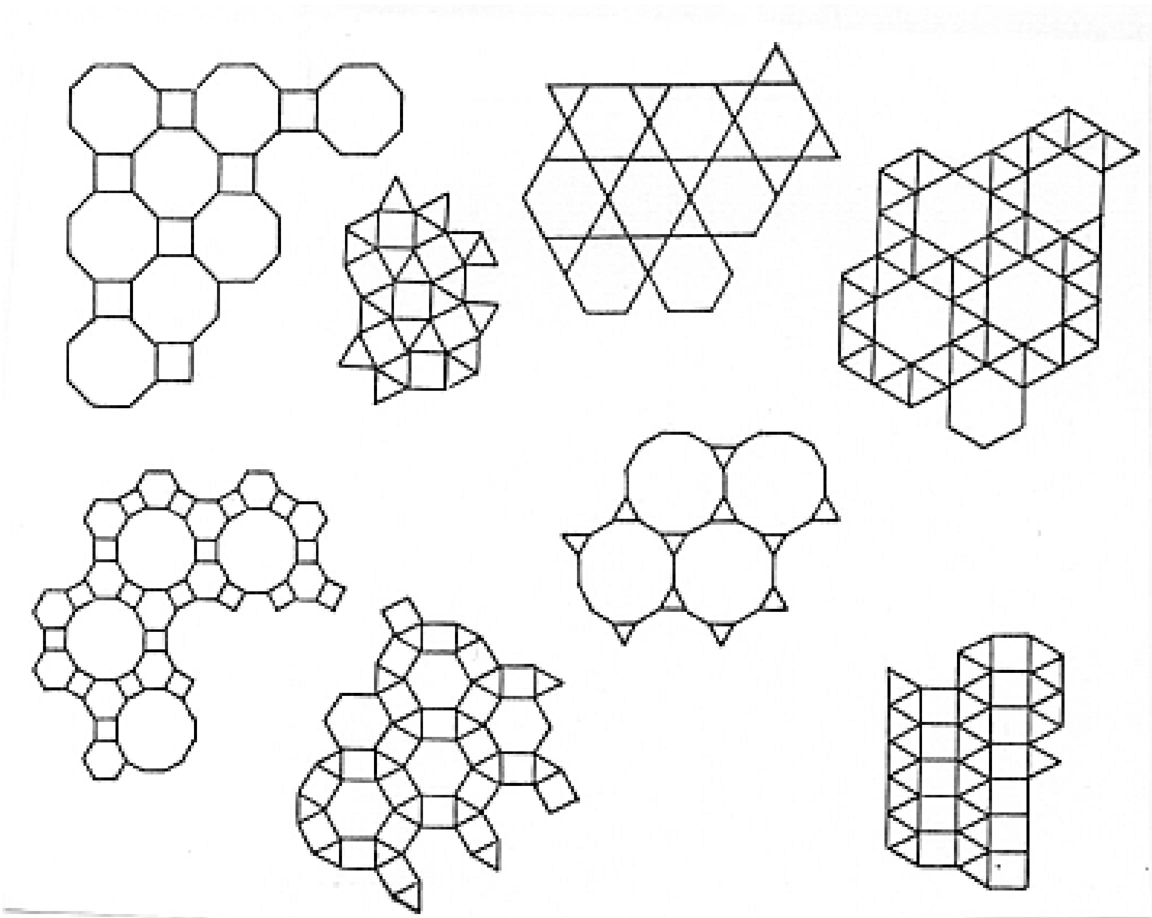

Illustration: Kappraff, p. 179

PLATONIC SOLIDS

Description: A Platonic solid has the property that each face is an identical regular polygon, and that the same number of polygons meet at each corner. Can you construct them all?

Comments: These are ancient objects of study. Plato (naturally!) is reputed to have investigated them, and they are discussed in Euclid's *Elements*, but they may be quite older.

Hints: There are only five Platonic solids:

1. Tetrahedron. Four equilateral triangles, three meeting at each corner.
2. Cube. Six squares, three meeting at each corner.
3. Octahedron. Eight equilateral triangles, four meeting at each corner.
4. Dodecahedron. Twelve regular pentagons, three meeting at each corner.
5. Icosahedron. Twenty equilateral triangles, five meeting at each corner.

References:

1. Holden, *Shapes, Space, and Symmetry*, Dover, 1971.
2. Wells, *The Penguin Dictionary of Curious and Interesting Geometry*, Penguin, 1991, pp. 187–188.

PLATONIC SOLIDS

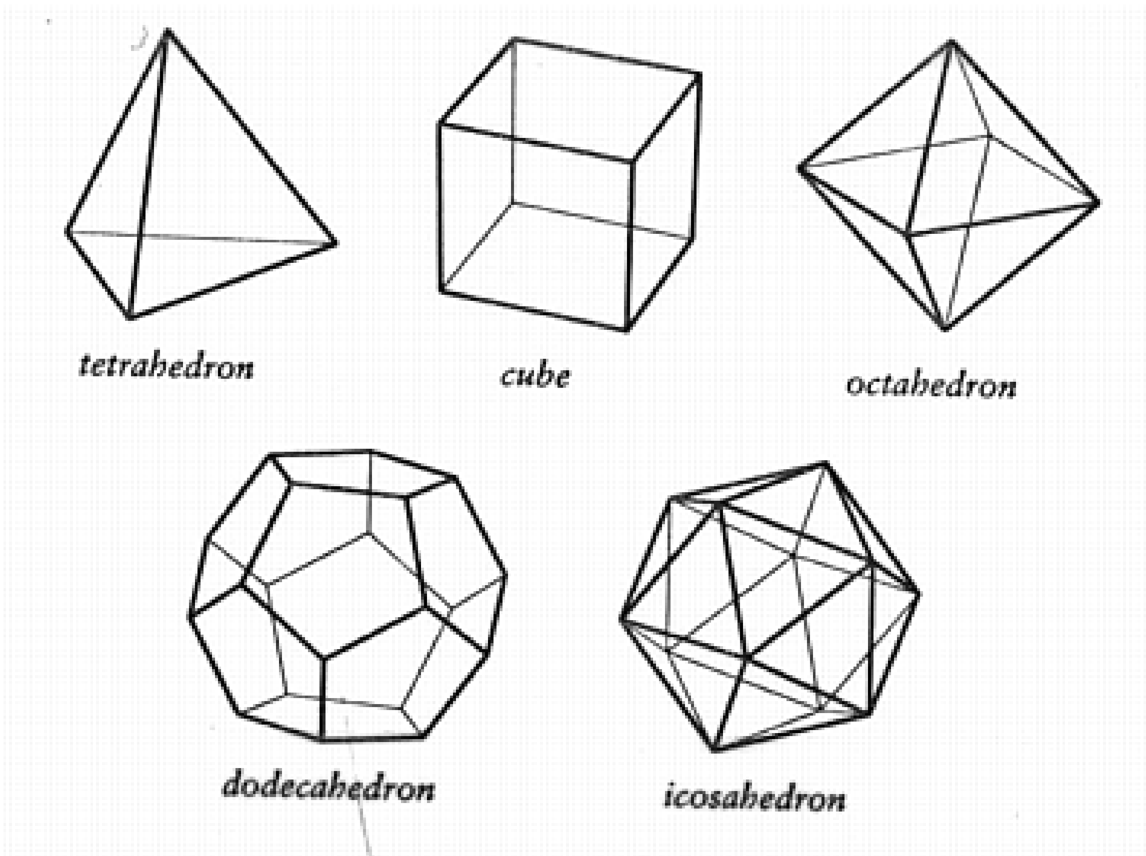

Illustration: Wells, p. 188

SEMIREGULAR SOLIDS

Description: A semiregular solid has the property that each face is a regular polygon, but that there are at least two different kinds of faces. In addition, the same pattern of faces must meet at each corner. How many can you construct?

Comments: According to Heron and Pappus, Archimedes wrote about these solids, but unfortunately the work has been lost. These are beautiful objects with very appealing symmetry.

Hints: Take two copies of any regular polygon and use them as the top and bottom of a "drum" ringed with squares. This is a prism. Now remove the squares and use a strip of equilateral triangles to form the sides of the drum. This is an antiprism. There is an infinite number of prisms and antiprisms—one for each choice of base polygon. Apart from these, there are exactly thirteen semiregular solids, also called Archimedean solids. One of them is made with hexagons and pentagons, and is the template for a soccer ball, as well as a representation of a recently discovered form of carbon, C_{60}, known as Buckminsterfullerene.

References:

1. Holden, *Shapes, Space, and Symmetry*, Dover, 1971.
2. Pearce and Pearce, *Polyhedra Primer*, Dale Seymour, 1978, pp. 55–66.
3. Wells, *The Penguin Dictionary of Curious and Interesting Geometry*, Penguin, 1991, pp. 6–8.

SEMIREGULAR SOLIDS

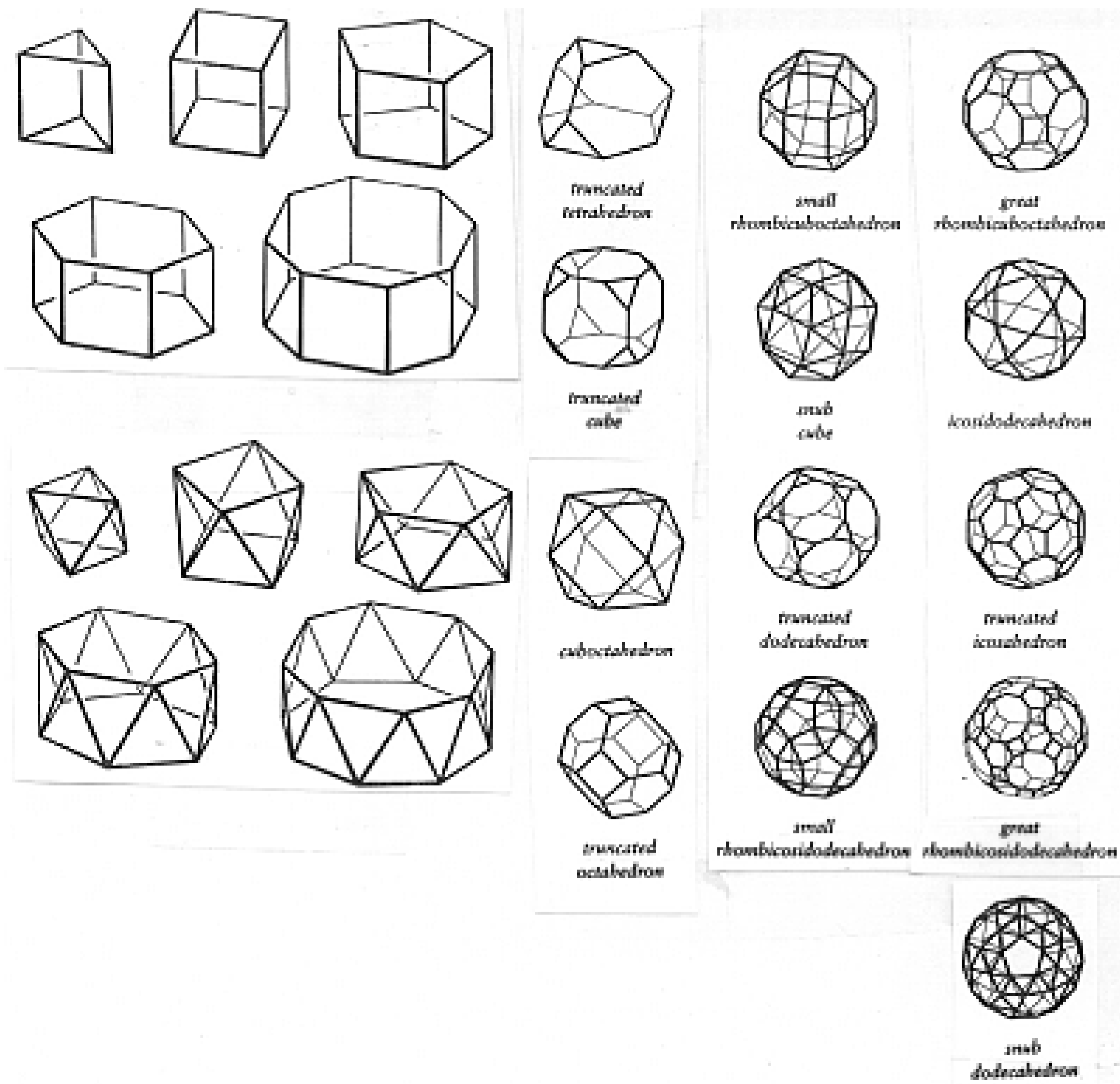

Illustration: Wells, pp. 6–7, Pearce-Pearce, p. 66

THE DELTAHEDRA

Description: A deltahedron is a polyhedron made solely from equilateral triangles. There is no requirement that the same number of triangles meet at every corner, but the object must be convex (i.e., it should have no indentations) and two triangles that meet along an edge must not lie in the same plane. How many can you construct?

Comments: It is conjectured that the name comes from the fact that the Greek letter Δ looks like a triangle.

Hints: Try to first understand why every deltahedron has an even number of faces. Experiment to see if you can determine the minimum and maximum number of faces. Curiously, one potential deltahedron is "missing"!

References:

1. Holden, *Shapes, Space, and Symmetry*, Dover, 1971, p. 3.
2. Pearce and Pearce, *Polyhedra Primer*, Dale Seymour, 1978, p. 67.

THE DELTAHEDRA

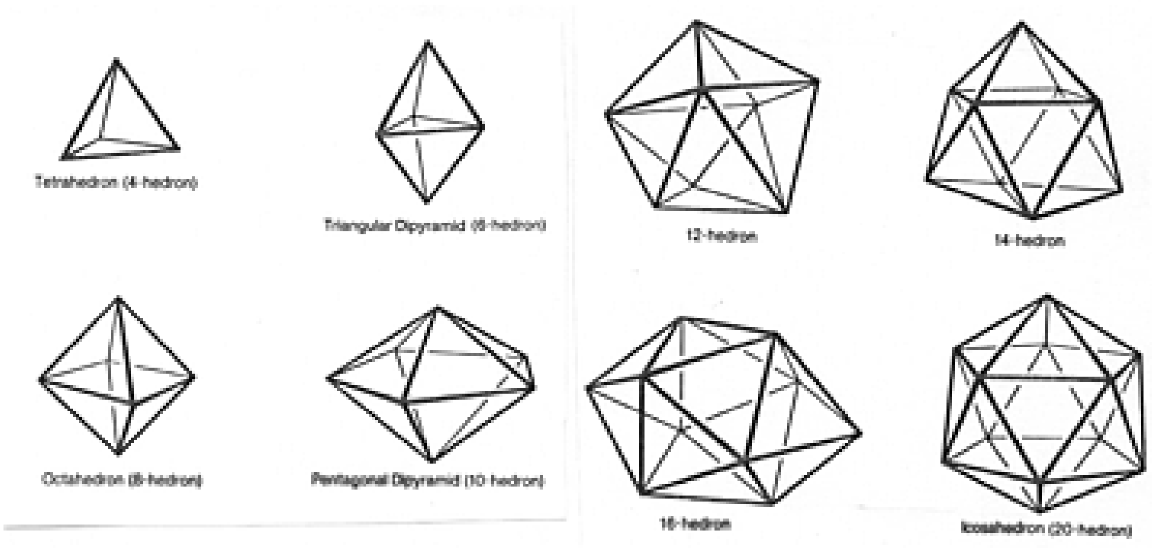

Illustration: Pearce-Pearce, p. 67