

Preliminary Examination in Discrete Mathematics

Outline:

Linear Programming and Combinatorial Optimization

1. Linear inequalities and polyhedra.
2. Simplex methods (primal, dual).
3. Revised simplex method.
4. Duality and complementary slackness.
5. Spanning trees.
6. Shortest path algorithms.
7. Max-flow/min cut; Edmonds-Karp specialization of the Ford-Fulkerson algorithm; Menger's Theorem.
8. Matching theory and applications.
9. Matroids; Greedy algorithm; Matroid polytope; Maximum cardinality matroid intersection.

Enumerative Combinatorics

1. Generating functions.
2. Stirling numbers of the first and second kind.
3. Permutations and permutation statistics.
4. q -analogues.
5. The twelvefold way.
6. Principle of inclusion-exclusion.
7. Partially ordered sets and lattices.
8. The fundamental theorem of distributive lattices.
9. The incidence algebra.
10. The Möbius inversion formula.
11. The Möbius function and computational techniques.
12. The Möbius algebra.
13. Semi-modular lattices and hyperplane arrangements.

14. The zeta polynomial.
15. Rank-selection.
16. R -labelings.
17. Eulerian posets.
18. Exponential generating functions.
19. The exponential formula.
20. Tree enumeration.
21. Lagrange inversion formula.

Suggested Texts:

1. B. Korte and J. Vygen, *Combinatorial Optimization: Theory and Algorithms*, Algorithms and Combinatorics 21, Springer, 2002. [Chapters 1–3, 6–10 (omitting 6.4, 7.3, 8.4–8.7, 9.3, 9.5, 10.4), 13 (omitting 13.7).]
2. D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, 1997. [Chapters 2–4.]
3. C. Lee, Linear Programming Lecture Notes, manuscript.
4. R. P. Stanley, *Enumerative Combinatorics, Vol. 1*, Cambridge Studies in Advanced Mathematics, 49, Cambridge University Press, 1997. [Chapters 1, 3 and 5.]

Additional References:

1. V. Chvátal, *Linear Programming*, Freeman and Co., 1983.
2. W. L. Winston, *Operations Research: Applications and Algorithms*, Duxbury Press, 1987.
3. G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, Wiley, 1999.
4. E. L. Lawler, *Combinatorial Optimization: Networks and Matroids*, Holt, Rinehart and Winston, 1976.
5. R. P. Stanley, *Enumerative Combinatorics, Vol. 2*, Cambridge Studies in Advanced Mathematics, 62, Cambridge University Press, 1999.

6. J. H. van Lint and R. M. Wilson, *A Course in Combinatorics*, second edition, Cambridge University Press, 2001.
7. H. S. Wilf, *Generatingfunctionology*, second edition, Academic Press, Inc., 1994.