

Essential Understandings in Geometry

Summarized from Two NCTM Books

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1 Middle School

- Big Idea 1. Behind every measurement formula lies a geometric result.
 - Essential Understanding 1a. Decomposing and rearranging provide a geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one.
 - Essential Understanding 1b. In addition to decomposing and rearranging, shearing provides another geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one.
- Big Idea 2. Geometric thinking involves developing, attending to, and learning how to work with imagery.
 - Essential Understanding 2a. Geometric images provide the content in relation to which properties can be noticed, definitions can be made, and invariances can be discerned.
 - Essential Understanding 2b. Symmetry provides a powerful way of working geometrically.
 - Essential Understanding 2c. Geometric awareness develops through practice in visualizing, diagramming, and constructing.
- Big Idea 3. A geometric object is a mental object that, when constructed, carries with it traces of the tool or tools by which it was constructed.

- Essential Understanding 3a. Tools provide new sources of imagery as well as specific ways of thinking about geometric objects and processes.
- Essential Understanding 3b. Geometric thinking turns tools into objects, and in geometry the process of turning an action undertaken with a tool into an object happens over and over again.
- Big Idea 4. Classifying, naming, defining, posing, conjecturing, and justifying are codependent activities in geometric investigation.
 - Essential Understanding 4a. Naming is not just about nomenclature: it draws attention to properties and objects of geometric interest.
 - Essential Understanding 4b. Definition can both generate and reflect structure: definitions are often dependent on a specific classification.
 - Essential Understanding 4c. Conjectures can emerge out of a problem-posing process that generates claims that need to be justified.

2 High School

- Big Idea 1. Working with diagrams is central to geometric thinking.
 - Essential Understanding 1a. A diagram is a sophisticated mathematical device for thinking and communicating.
 - Essential Understanding 1b. A diagram is a “built” geometric artifact, with both a history—a narrative of successive construction—and a purpose.
 - Essential Understanding 1c. A diagram is not a picture. It needs to be interpreted: learning how to read a diagram can be like learning a new language.
- Big Idea 2. Geometry is about working with variance and invariance, despite appearing to be about theorems.
 - Essential Understanding 2a. Underlying any geometric theorem is an invariance—something that does not change while something else does.
 - Essential Understanding 2b. Invariances are rare and can be appreciated only when they emerge out of much greater variation.
 - Essential Understanding 2c. Examining the possible variations of an invariant situation can lead to new conjectures and theorems.

- Essential Understanding 2d. Geometry is a dynamic study, even if it often appears to be static.
- Big Idea 3. Working with and on definitions is central to geometry.
 - Essential Understanding 3a. Geometric objects can have different definitions. Some are better than others, and their worth depends both on context and values.
 - Essential Understanding 3b. Definitions in geometry are of two distinct types: definition by genesis (how you can create the object) and definition by property (how you can characterize the object in terms of certain features).
 - Essential Understanding 3c. Building definitions requires moving back and forth between the verbal and the visual.
- Big Idea 4. A written proof is the endpoint of the process of proving.
 - Essential Understanding 4a. Empirical verification is an important part of the process of proving, but it can never, by itself, constitute a proof.
 - Essential Understanding 4b. Counterexamples are important: individual instances can disprove a conjecture, but they can also lead to modified conjectures.
 - Essential Understanding 4c. Behind every proof is a proof idea.
 - Essential Understanding 4d. Geometry uses a wide variety of kinds of proofs.