

Mathematics II Topics

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1 Understand Similarity in Terms of Similarity Transformations

1. *A framework for geometry incorporating transformations, including proving the triangle congruence theorems.*
2. Verify experimentally the properties of dilations given by a center and a scale factor. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

GeoGebra and SketchUp explorations.

SketchUp: Create shape, form into group, make a copy with the Move tool, mark lengths with Dimension tool, use Scale tool.

3. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
4. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

2 Prove Geometric Theorems

6–8 Big Idea 2. Geometric thinking involves developing, attending to, and learning how to work with imagery.

6–8 Essential Understanding 2a. Geometric images provide the content in relation to which properties can be noticed, definitions can be made, and invariances can be discerned.

6–8 Essential Understanding 2b. Symmetry provides a powerful way of working geometrically.

6–8 Essential Understanding 2c. Geometric awareness develops through practice in visualizing, diagramming, and constructing.

6–8 Big Idea 3. A geometric object is a mental object that, when constructed, carries with it traces of the tool or tools by which it was constructed.

6–8 Essential Understanding 3a. Tools provide new sources of imagery as well as specific ways of thinking about geometric objects and processes.

6–8 Essential Understanding 3b. Geometric thinking turns tools into objects, and in geometry the process of turning an action undertaken with a tool into an object happens over and over again.

6–8 Big Idea 4. Classifying, naming, defining, posing, conjecturing, and justifying are codependent activities in geometric investigation.

6–8 Essential Understanding 4a. Naming is not just about nomenclature: it draws attention to properties and objects of geometric interest.

6–8 Essential Understanding 4b. Definition can both generate and reflect structure: definitions are often dependent on a specific classification.

6–8 Essential Understanding 4c. Conjectures can emerge out of a problem-posing process that generates claims that need to be justified.

9–12 Big Idea 1. Working with diagrams is central to geometric thinking.

9–12 Essential Understanding 1a. A diagram is a sophisticated mathematical device for thinking and communicating.

9–12 Essential Understanding 1b. A diagram is a “built” geometric artifact, with both a history—a narrative of successive construction—and a purpose.

9–12 Essential Understanding 1c. A diagram is not a picture. It needs to be interpreted: learning how to read a diagram can be like learning a new language.

9–12 Big Idea 2. Geometry is about working with variance and invariance, despite appearing to be about theorems.

9–12 Essential Understanding 2a. Underlying any geometric theorem is an invariance—something that does not change while something else does.

9–12 Essential Understanding 2b. Invariances are rare and can be appreciated only when they emerge out of much greater variation.

9–12 Essential Understanding 2c. Examining the possible variations of an invariant situation can lead to new conjectures and theorems.

9–12 Essential Understanding 2d. Geometry is a dynamic study, even if it often appears to be static.

9–12 Big Idea 3. Working with and on definitions is central to geometry.

9–12 Essential Understanding 3a. Geometric objects can have different definitions. Some are better than others, and their worth depends both on context and values.

9–12 Essential Understanding 3b. Definitions in geometry are of two distinct types: definition by genesis (how you can create the object) and definition by property (how you can characterize the object in terms of certain features).

9–12 Essential Understanding 3c. Building definitions requires moving back and forth between the verbal and the visual.

9–12 Big Idea 4. A written proof is the endpoint of the process of proving.

9–12 Essential Understanding 4a. Empirical verification is an important part of the process of proving, but it can never, by itself, constitute a proof.

9–12 Essential Understanding 4b. Counterexamples are important: individual instances can [sometimes] disprove a conjecture, but they can also lead to modified conjectures. [Consider these conjectures: (1) Every rectangle is a square. (2) There exist three points in the plane equidistant from each other.]

9–12 Essential Understanding 4c. Behind every proof is a proof idea.

9–12 Essential Understanding 4d. Geometry uses a wide variety of kinds of proofs.

1. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should

be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

2. Prove theorems about lines and angles. Theorems include:

(a) Vertical angles are congruent.

Via angle measure and via rigid motions.

(b) When a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

Via dilation.

(c) Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Two directions need to be proved.

(d) Points on an angle bisector are exactly those equidistant from the angle's sides.

Two directions need to be proved.

(e) The perpendicular bisectors of the sides of a triangle are concurrent.

Uses both a theorem and its converse. Leads to circumscribed circle.

(f) The angle bisectors of a triangle are concurrent.

Uses both a theorem and its converse. Leads to inscribed circle.

3. Prove theorems about triangles. Theorems include:

(a) Measures of interior angles of a triangle sum to 180° .

Via parallel line and transversals.

(b) Base angles of isosceles triangles are congruent.

Via reflection.

(c) The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.

Via dilation.

(d) The medians of a triangle meet at a point.

Coordinate proof, using locating points that divide segments in desired ratios. Area proof. Proof by shearing.

4. Prove theorems about parallelograms. Theorems include:

(a) If a quadrilateral is a parallelogram then opposite sides are congruent.

- (b) If a quadrilateral is a parallelogram then opposite angles are congruent.
- (c) If a quadrilateral is a parallelogram then the diagonals bisect each other
- (d) And conversely, if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- (e) If a quadrilateral is a rectangle, then it is a parallelogram with congruent diagonals.
- (f) If a quadrilateral is a parallelogram with congruent diagonals, then it is a rectangle.

May provide good context for flow diagram proofs.

Use hypotheses to construct a figure (e.g., a quadrilateral with bisecting diagonals), and test the construction by dragging. Illustrates many of the Big Ideas.

3 Prove Theorems Involving Similarity

1. Prove theorems about triangles. Theorems include:
 - (a) A line parallel to one side of a triangle divides the other two proportionally
 - (b) And conversely, if a line divides two sides of a triangle proportionally, then the line is parallel to the third side.
 - (c) The Pythagorean Theorem proved using triangle similarity.
2. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Look for examples in the place of the community.

Estimate height of person from a photo of him/her holding an object of known size. Drive-in movie problem. Astronomical measurement with a coin.

Navigating, Chapter 3: Scale Factors, Basic Dilations, Coordinate Connections, Multiple Transformations, Shadowy Measurements, Field of Vision. See back of book for blackline masters and solutions.

Focus: Field of Vision problem, p.14.

Yearbook: Chapter 5, Prairie Plants: Exploring Fractal Forms in Nature.

4 Use Coordinates to Prove Simple Geometric Theorems Algebraically

1. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Concurrence of medians of a triangle. Animation of straight-line motion.

5 Define Trigonometric Ratios and Solve Problems Involving Right Triangles

1. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
2. Explain and use the relationship between the sine and cosine of complementary angles.
3. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Look for examples in the place of the community.

Demonstrate iPad apps Easy Measure and Theodolite.

6 Prove and Apply Trigonometric Identities

1. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle. In this course, limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.

Introduce the unit circle and identities via reflections and rotations. Use for planetary animations.

7 Understand and Apply Theorems About Circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii and chords. Include:
 - (a) The relationship between central, inscribed and circumscribed angles.
 - (b) Inscribed angles on a diameter are right angles.
 - (c) The radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Can be approached by a limiting process of inscribed angles.

3. Construct the inscribed and circumscribed circles of a triangle.
4. Prove properties of angles for a quadrilateral inscribed in a circle.
5. Construct a tangent line from a point outside a given circle to the circle.

8 Find Arc Lengths and Areas of Sectors of Circles.

1. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality.
2. Derive the formula for the area of a sector. Emphasize the similarity of all circles.
3. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure.

9 Translate Between the Geometric Description and the Equation for a Conic Section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem.

2. Complete the square to find the center and radius of a circle given by an equation.
Relate this to formulas for translations and scalings, even for general objects.
3. Derive the equation of a parabola given the focus and directrix.
Extend to ellipses and hyperbolas.

10 Use Coordinates to Prove Simple Geometric Theorems Algebraically

1. Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.
2. Prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.
3. Include simple proofs involving circles.

Connect midpoints of quadrilateral.

11 Explain Volume Formulas and Use Them to Solve Problems

6–8 Big Idea 1. Behind every measurement formula lies a geometric result.

1. Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k , its area is k^2 times the area of the first.
Scale a figure and a superimposed grid.

2. Volumes of solid figures scale by k^3 under a similarity transformation with scale factor k .

Scale a multilink cube building. Show scaling effects in formulas.

3. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Focus: Developing a Formula for Volume, p.63.

Include parallelepipeds (shears and Cavalieri) and spheres (Cavalieri).

4. Use dissection arguments, Cavalieri's principle and informal limit arguments.

For spheres, related volume and surface area, with analogy to area and circumference of circles. 6–8 Essential Understanding 1a. Decomposing and rearranging provide a geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one. See example of pyramids (Figure 1.13 and 1.21).

6–8 Essential Understanding 1b. In addition to decomposing and rearranging, shearing provides another geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one. See example of parallelograms and triangles (Figures 1.14–1.16), and parallelepipeds (Figure 1.19).

5. Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.

Maximizing volume given surface area. Volume (and surface area) of "dixie cup".

Look for problems in the place of the community.