MA109, Activity 1: Basic Equations (Section 1.1, pp. 74-80)   Date: ______

Today's Goal:	Equations are the basic mathematical tool for solving real-world problems. We introduce them and we learn how to solve some special classes.
Assignments:	**Homework (Sec. 1.1):** # 1, 4, 7, 15, 18, 20, 23, 28, 33, 40, 49, 55, 62, 88 (pp. 80-83).

An **equation** is a statement that two mathematical expressions are equal. For instance,

$$4^3 - 2 \cdot 4^2 = 32.$$

Most equations that we study in Algebra contain variables. For example,

$$x^3 - 2x^2 = 32.$$

Given an equation in the variable (let's say) x, the **goal** is to find the values of x that make the equation true; these values are called the **solutions** or **roots** of the equation, and the process of finding the solutions is called **solving the equation**.

Example 1: Using the previous terminology, we say that $x =$ 4 is a solution (or root) of the equation:

$$x^3 - 2x^2 = 32$$

Example 2: Determine whether the given value of x is a solution of the equation:

$$1 - [2 - (3 - x)] = 4x - (6 + x)$$

(a) $x = 2$

Left $= 1-[2-(3-2)]$
$= 1-[2-1]$
$= 1-1$
$= 0$
Right $= 4\cdot 2-(6+2) = 8-8=0$
Since Left = Right, x=2
is a solution

(b) $x = 4$

Left $= 1-[2-(3-4)]$
$= 1-[2-(-1)]$
$= 1-3$
$= -2$

Right $= 4\cdot 4-(6+4)$
$= 16-10$
$= 6$

Left $\neq$ Right, so x=4 is not
a solution

Two equations with exactly the same solutions are called **equivalent equations**. To solve an equation, we try to find a simpler, equivalent equation in which the variable stands alone on one side of the equal sign. In many cases, this is achieved by using the following properties.

Properties of Equations

1. $A = B \iff A + C = B + C$
2. $A = B \iff AC = BC \quad (C \neq 0)$

Types of Equations

▶ **Linear Equations:**

A **linear equation** (or first-degree equation) in one variable is an equation equivalent to one of the form:

$$ax + b = 0,$$

where a and b are real numbers and x is the variable.

The unique solution of the linear equation $ax + b = 0$ is:

$$x = -\frac{b}{a}$$

(provided $a \neq 0$)

Example 3: The given equations are linear or equivalent to linear equations. Solve these equations:

- $5t - 13 = 12 - 5t$

$10t - 13 = 12$

$10t = 25$

$t = \frac{25}{10}$

$t = 2.5$

- $\frac{2}{3}y + \frac{1}{2}(y-3) = \frac{y+1}{4}$

common multiple $= 12$

$12 \cdot \frac{2}{3}y + 12 \cdot \frac{1}{2}(y-3) = 12 \cdot \frac{y+1}{4}$

$8y + 6(y-3) = 3(y+1)$

$14y - 18 = 3y + 3$

$11y = 21$

$\boxed{y = 21/11}$

- $(x+3)^2 = (x-1)^2 - 8$

$\cancel{x^2} + 6x + 9 = (\cancel{x^2} - 2x + 1) - 8$

$8x + 9 = -7$

$8x = -16$

$x = \frac{-16}{8}$

$x = -2$

- $\frac{1}{x+3} + \frac{5}{x^2-9} = \frac{2}{x-3}$

multiply by $(x+3)(x-3) = x^2 - 9$

$\frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}} + \frac{5\cancel{(x^2-9)}}{\cancel{x^2-9}} = \frac{2(x+3)\cancel{(x-3)}}{\cancel{x-3}}$

$x - 3 + 5 = 2(x+3)$

$x + 2 = 2x + 6$

$-4 = x$

▶ Solving Equations Using Radicals:

Now, we consider basic equations that can be simplified into the form

$$X^n = a, \qquad n \geq 1.$$

These equations can be solved by taking radicals of both sides of the equation. We can also solve simple equations involving a fractional power of the variable.

Solving a Simple n^{th} Degree Equation

The real solutions of the equation $X^n = a$ are:

- $X = \sqrt[n]{a}$, if n is odd
- $X = \pm\sqrt[n]{a}$, if n is even and $a \geq 0$
- no real solutions, if n is even and $a < 0$

Example 4: The given equations involve a power of a variable. Find all real solutions of these equations:

- $x^2 = 49$

$x = \pm\sqrt{49}$

$x = \pm 7$

- $x^2 + 16 = 0$

$x^2 = -16$

no real solutions

- $3(z-5)^2 - 15 = 0$

$3(z-5)^2 = 15$

$(z-5)^2 = 5$

$z - 5 = \pm\sqrt{5}$

$z = 5 \pm \sqrt{5}$

- $\sqrt[3]{y} = 5$

$y^{1/3} = 5$

$(y^{1/3})^3 = (5)^3$

$y = 125$

- $x^4 - 16 = 0$

$x^4 = 16$

$x = \pm \sqrt[4]{16}$

$= \pm \sqrt[4]{2^4}$

$= \pm 2$

- $\frac{x^5}{15} = 2$

$x^5 = 30$

$x = \sqrt[5]{30}$

(Alternatively, factor the polynomial $x^4 - 16$ above and obtain your solutions from such factorization.)

Example 5: The average daily food consumption F of an herbivorous mammal with body weight w, where both F and w are measured in pounds, is given approximatively by the equation

$$F = 0.3w^{3/4}.$$

Find the weight w of an elephant who consumes 300 lb of food per day.

$300 = 0.3\,\omega^{3/4}$

$\omega^{3/4} = \frac{300}{0.3} = 1000$

$\omega = 1000^{4/3}$

► **Solving for One Variable in Terms of Others:** Many formulas in the sciences involve several variables, and it is often necessary to express one of the variables in terms of the others.

Example 6: Solve the given equation for the indicated variable.

- $F = G\frac{mM}{r^2}$; for m

$Fr^2 = GmM$

$= (GM)m$

$\frac{Fr^2}{GM} = m$

- $\frac{a+1}{b} = \frac{a-1}{b} + \frac{b+1}{a}$; for a

$a\cancel{b}\,\frac{a+1}{\cancel{b}} = ab\cdot\frac{a-1}{\cancel{b}} + \cancel{a}b\cdot\frac{b+1}{\cancel{a}}$

$a(a+1) = a(a-1) + b(b+1)$

$\cancel{a^2} + a = \cancel{a^2} - a + b(b+1)$

$2a = b(b+1)$

$a = b(b+1)/2$

- $\frac{ax+b}{cx+d} = 2$; for x

$ax + b = 2(cx + d)$

$ax + b = 2cx + 2d$

$ax - 2cx = 2d - b$

$(a - 2c)x = 2d - b$

$x = \frac{2d-b}{a-2c}$

- $a^2x + (a-1) = (a+1)x$; for x

$a^2x - (a+1)x = -(a-1)$

$(a^2 - (a+1))x = 1 - a$

$x = \frac{1-a}{a^2-a-1}$