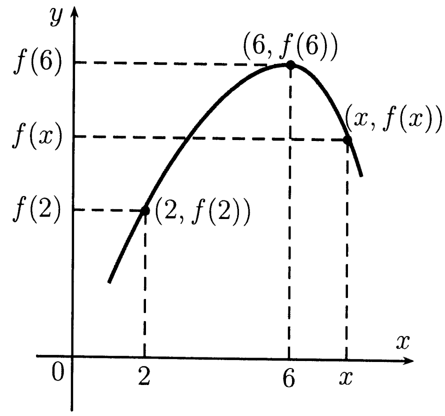
MA109, Activity 12: Graphs of Functions (Section 3.2, pp. 224-232) **Date:** ____________

`Today's Goal:` The graph of a function is the most important way to visualize a function. It gives a picture of the behavior or 'life history' of the function. We can read the value of $f(x)$ from the graph as being the height of the graph above the point x.

`Assignments:` **Homework (Sec. 3.2):** #1,5,8,13,17,20,24,25,39,44,53 (pp. 233-236).

► Graphing Functions:



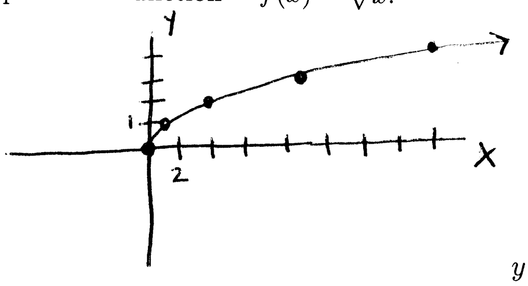
If f is a function with domain A, then the graph of f is the set of ordered pairs

$$\textbf{graph of } f = \{(x, f(x)) \mid x \in A\}.$$

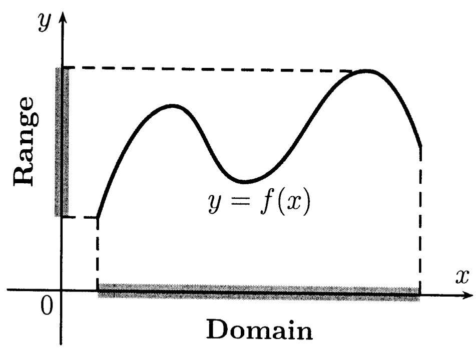
In other words, the graph of f is the set of all points (x, y) such that $y = f(x)$; that is, the graph of f is the graph of the equation $y = f(x)$.

Example 1: Sketch the graph of the function $f(x) = \sqrt{x}$.

x	f(x)
0	0
1	$\sqrt{1}=1$
4	$\sqrt{4}=2$
9	$\sqrt{9}=3$
16	$\sqrt{16}=4$

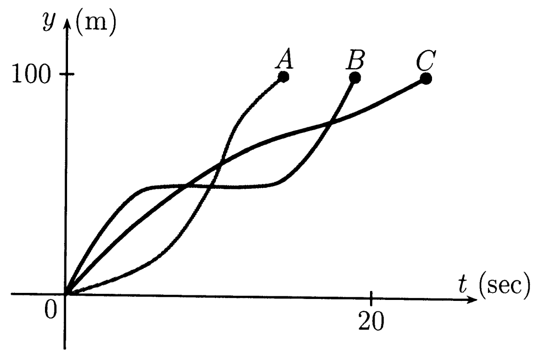


► Obtaining Information from the Graph of a Function:



The values of a function are represented by the height of its graph above the x-axis. So, we can read off the values of a function from its graph.
In addition, the graph of a function helps us picture the domain and range of the function on the x-axis and y-axis as shown in the picture:

Example 2 (Hurdle Race): Three runners compete in a 100-meter hurdle race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race? What do you think happened to runner B?



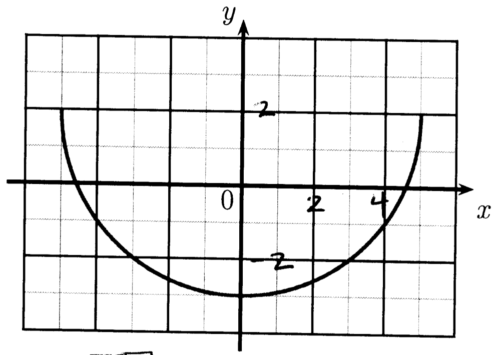
Runner A won the race.

All three runners finished the race.

Runner B rested about halfway through the race, perhaps he had an injury.

Example 3: The picture shows the graph of

$$g(x) = 2 - \sqrt{25 - x^2}.$$



From the graph, find the domain and range of g.

domain : $[-5, 5]$

range : $[-3, 2]$

Find $g(4)$ and $g(-2)$.

$g(4) = 2 - \sqrt{25-(4)^2} = 2 - \sqrt{25-16} = 2 - \sqrt{9} = 2 - 3 = \boxed{-1}$

$g(-2) = 2 - \sqrt{25-(-2)^2} = 2 - \sqrt{25-4} = \boxed{2 - \sqrt{21}}$

If $g(x) = -2$, what is x?

$-2 = 2 - \sqrt{25-x^2}$

$-4 = -\sqrt{25-x^2}$

$4 = \sqrt{25-x^2}$

$16 = 25 - x^2$

$-9 = -x^2$

$x^2 = 9$

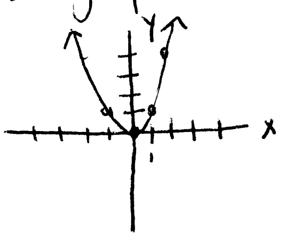
$\boxed{x = \pm 3}$

▶ **Graphing Piecewise Defined Functions:**

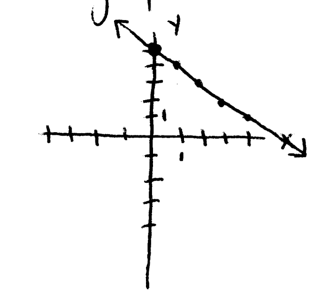
A piecewise defined function is defined by different formulas on different parts of its domain. As you might expect, the graph of such a function consists of separate pieces.

Example 4: Sketch the graph of the piecewise function $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ 5 - x & \text{if } x > 2 \end{cases}$

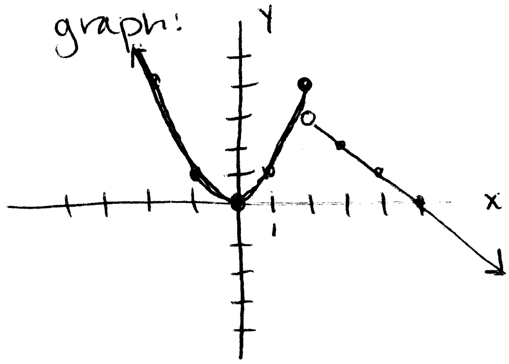
note $f(x) = x^2$ has graph:



and $f(x) = 5 - x$ has graph:



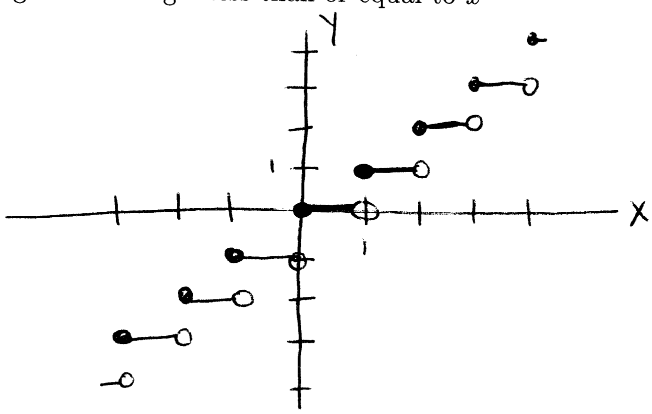
so, $f(x)$ as defined above has graph:



Example 5: **Step functions** occur frequently in real-world examples. The **greatest integer function** is one of them; sketch its graph:

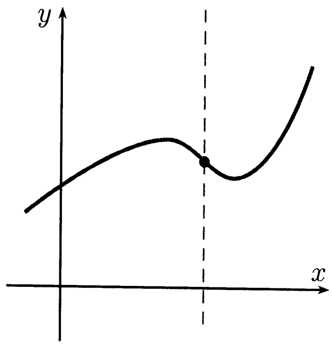
$$[\![x]\!] = \text{greatest integer less than or equal to } x$$

Note:
every number between 0 and 1 gets mapped to 0, every # between 1 and 2 gets mapped to 1, every # between 2 and 3 gets mapped to 2, and so on...

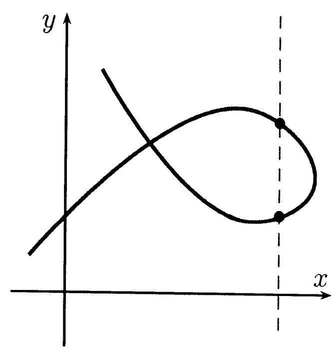


The graph of a function is a curve in the xy-plane. But the question arises: Which curves in the xy-plane are graphs of functions?

► **The Vertical Line Test:**
A curve in the coordinate plane is the graph of a function if and only if no vertical line intersects the curve more than once.

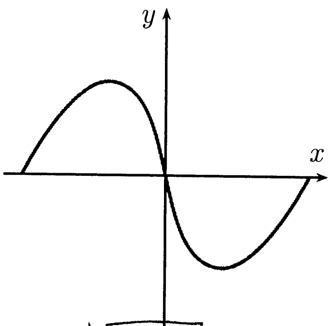


Graph of a function

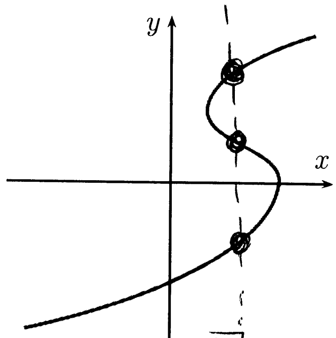


Not a graph of a function

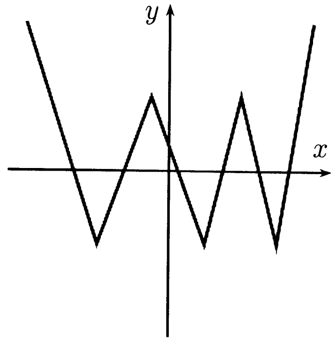
Example 6: Determine which of the curves drawn below is the graph of a function of x.



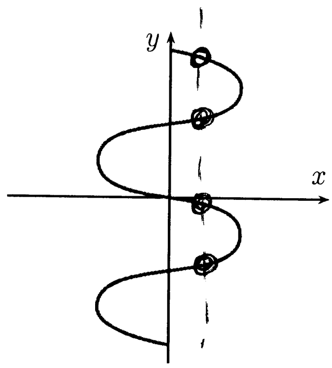
YES



NO



YES



NO

► **Equations That Define Functions:**

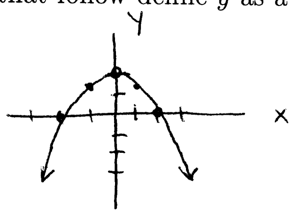
Not every equation involving two variables (say x and y) defines one of the variables as a function of the other (say y as a function of x).

Example 7: Which of the equations that follow define y as a function of x?

$x^2 + 2y = 4$

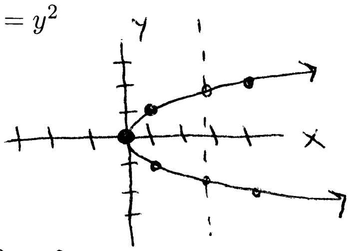
$2y = -x^2 + 4$

$y = -\frac{1}{2}x^2 + 2$



Yes, this defines y as a function of x.

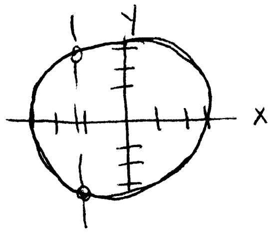
$x = y^2$



No, this does not define y as a function of x by the vertical line test.

$x^2 + y^2 = 9$

Recall: this is the equation for a circle with center (0,0) and radius 3.



No, this does not define y as a function of x by the vertical line test.

Some Functions and Their Graphs: Check the table on page 232 of our textbook for a list of the graphs of the functions that we will frequently encounter from now on in the course. The list includes: Linear functions, power functions, root functions, reciprocal functions, absolute value functions, and step functions.