MA109, Activity 13: Increasing and Decreasing Functions; (Section 3.3, pp. 237-243)
Average Rate of Change

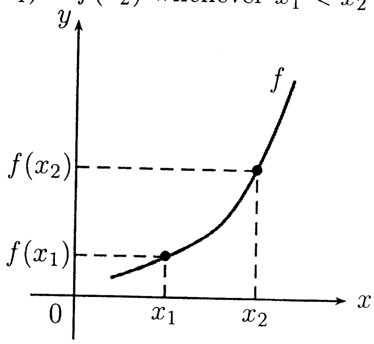
Date: ____________

Today's Goal:	Functions are often used to model changing quantities. We learn how to determine if a function is increasing or decreasing, and how to find the rate at which its values change as the variable change.
Assignments:	**Homework (Sec. 3.3):** # 1, 4, 13, 15, 17, 19, 22, 31, 33 (pp. 244-246).

► **Increasing and Decreasing Functions:**

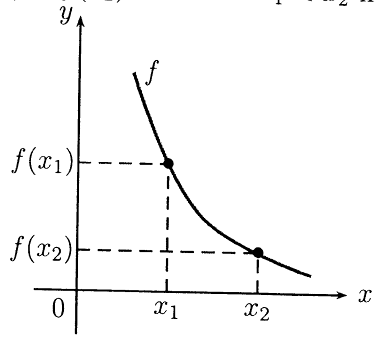
A function f is said to be increasing when its graph rises and decreasing when its graph falls. More precisely, we say that:

f is **increasing** on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

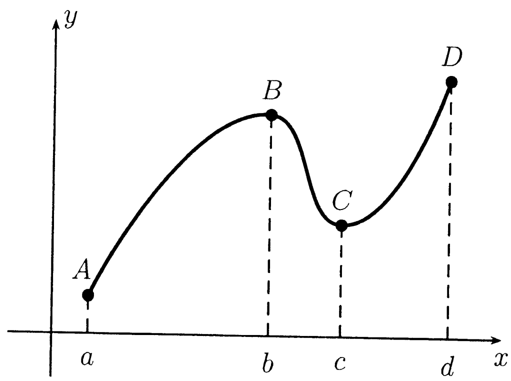


f is increasing

f is **decreasing** on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.



f is decreasing



f is increasing on the intervals $[a, b]$ and $[c, d]$
f is decreasing on the interval $[b, c]$.

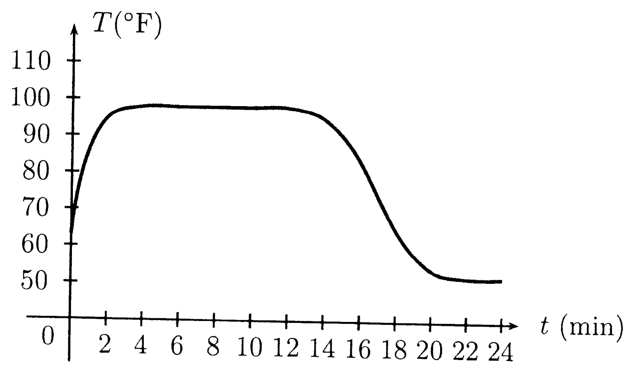
Example 1:

The picture shows a rough graph of the temperature T of the water from a faucet as a function of the time t that has elapsed since the faucet was turned on.

On which intervals is this function increasing?

Where is it decreasing?

Where is it constant?



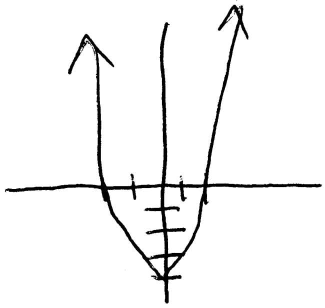
increasing : $[0, 4]$

constant : $[4, 12]$, $[20, 24]$

decreasing : $[12, 20]$

Example 2:

Graph the function $f(x) = x^2 - 4$. State the intervals on which f is increasing and on which f is decreasing.



decreasing on $(-\infty, 0]$
increasing on $[0, \infty)$

► **Average Rate of Change:**

We are all familiar with the concept of speed: If you drive a distance of 120 miles in two hours, then your average speed, or rate of travel, is $120/2 = 60$ miles per hour. In other words, the average speed is equal to the ratio of the distance traveled over the time elapsed:

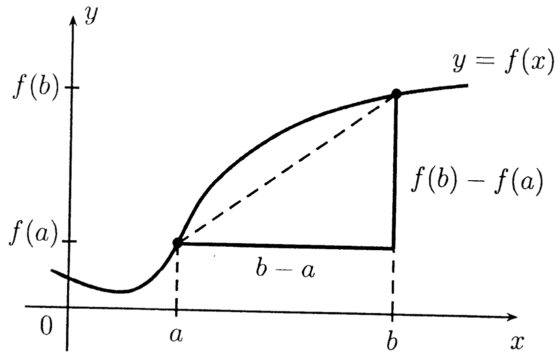
$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}.$$

Finding average rates of change is important in many contexts. For instance, we may be interested in knowing how quickly the air temperature is dropping as a storm approaches, or how fast revenues are increasing from the sale of a new product.

The **average rate of change** of the function $y = f(x)$ between $x = a$ and $x = b$ is

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$

The average rate of change is the slope of the **secant line** between $x = a$ and $x = b$ on the graph of f, that is, the line that passes through $(a, f(a))$ and $(b, f(b))$.



Example 3:

Consider the function $h(x) = 3x - 2$. Find the average rate of change of the function between $x = 2$ and $x = 4$.

$$\frac{h(4) - h(2)}{4 - 2} = \frac{h(4) - h(2)}{2} = \frac{3(4) - 2 - 3(2) + 2}{2}$$

$$= \frac{12 - 2 - 6 + 2}{2} = \frac{6}{2} = 3 = \text{slope of } f(x) = 3x - 2$$

Example 4:

this the difference quotient

Consider the function $g(x) = \frac{2}{x}$. Find the average rate of change of the function between $x = a$ and $x = a + h$.

$$\frac{g(a+h)-g(a)}{a+h-a} = \frac{g(a+h)-g(a)}{h} \overset{\text{step 2}}{=} \frac{\frac{-2h}{a(a+h)}}{h} = \frac{-2}{a(a+h)}$$

step 1 $g(a) = \frac{2}{a}$

$g(a+h) = \frac{2}{a+h}$

$$g(a+h)-g(a) = \frac{2}{a+h} - \frac{2}{a} = \frac{2a}{a(a+h)} - \frac{2(a+h)}{a(a+h)}$$

$$= \frac{-2h}{a(a+h)}$$

Example 5:

If an object is dropped from a tall building, then the distance it has fallen after t seconds is given by the function $d(t) = 16t^2$. Find its average speed (average rate of change) over the following intervals:

(a) between $t = 1$ and $t = 5$ seconds;

$$\frac{d(5)-d(1)}{5-1} = \frac{16(5)^2-16(1)^2}{4}$$

$$= \frac{400-16}{4}$$

$$= \frac{384}{4} = 96 \text{ m}$$

(b) between $t = a$ and $t = a + h$ seconds.

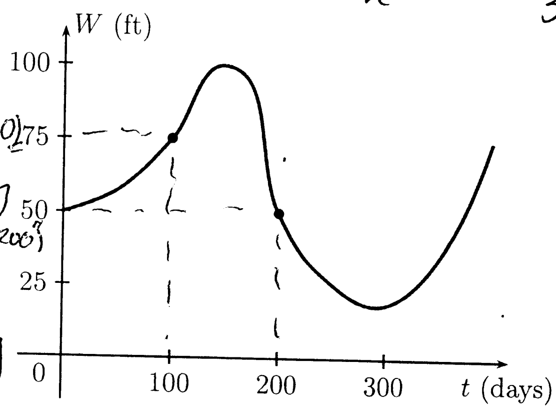
$$\frac{d(a+h)-d(a)}{a+h-a} = \frac{16(a+h)^2-16(a)^2}{h}$$

$$= \frac{16a^2+32ah+16h^2-16a^2}{h}$$

$$= \frac{32ah+16h^2}{h} = 32a+16h$$

Example 6:

The graph shows the depth of water W in a reservoir over a one-year period, as a function of the number of days t since the beginning of the year.



(a) Determine the intervals on which the function W is increasing and on which it is decreasing.

(b) What was the average rate of change of W between $t = 100$ and $t = 200$?

(a) increasing $[0, 150]$, $[300, 365]$

decreasing $[150, 300]$

(b) $$\frac{W(200)-W(100)}{200-100} = \frac{50-75}{100} = \frac{-25}{100}$$

Example 7:

By looking at Example 3, what can you conclude about the Rate of Change of a linear function $y = mx + b$ between any two points x_0 and x_1?

$f(x) = mx + b$ = any linear function

$$\frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{mx_1+b-mx_0-b}{x_1-x_0} = \frac{m(x_1-x_0)}{(x_1-x_0)} = m$$