MA109, Activity 14: Transformations of Functions (Section 3.4, pp. 247-255) Date: _____

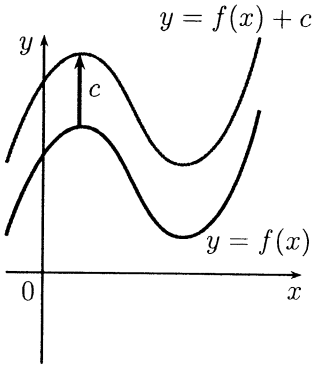
Today's Goal: We study how certain transformations of a function affect its graph. This will give us a better understanding of how to graph functions. The transformations we study are shifting, reflecting, and stretching.

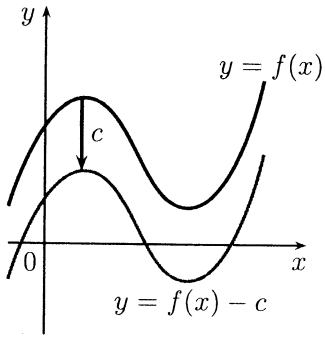
Assignments: **Homework (Sec. 3.4):** # 1, 3, 4, 7, 11, 12, 14, 15, 17, 25, 35, 43, 53, 64, 68 (pp. 255-259).

▶ **Vertical Shifting:** Suppose $c > 0$.

To graph $y = f(x) + c$, shift the graph of $y = f(x)$ UPWARD c units.

To graph $y = f(x) - c$, shift the graph of $y = f(x)$ DOWNWARD c units.

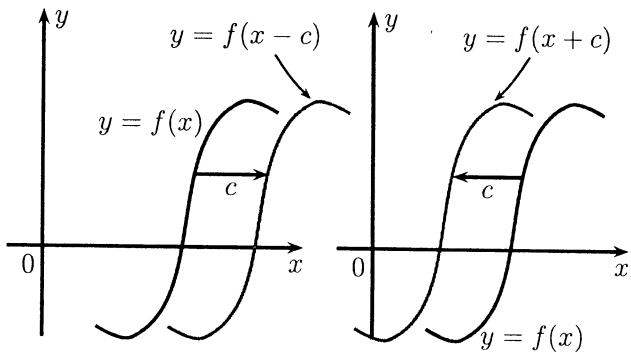




▶ **Horizontal Shifting:** Suppose $c > 0$.

To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the RIGHT c units.

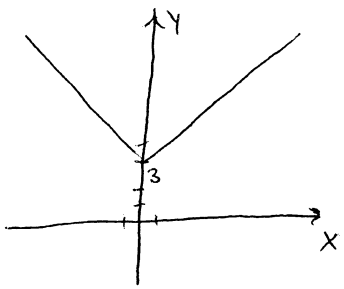
To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the LEFT c units.



Example 1: Use the graph of $y = |x|$ to sketch the graphs of the following functions:

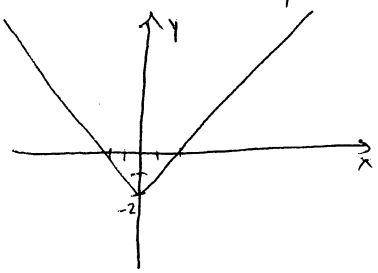
$y = |x| + 3$

Shift up by 3

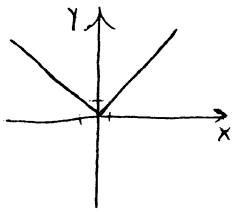


$y = |x| - 2$

Shift down by 2



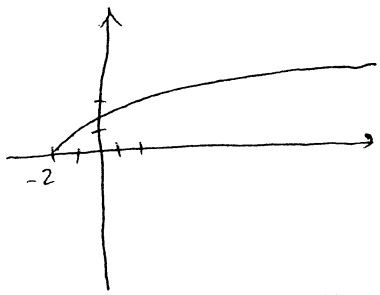
Recall $y = |x|$



Example 2: Use the graph of $y = \sqrt{x}$ to sketch the graphs of the following functions:

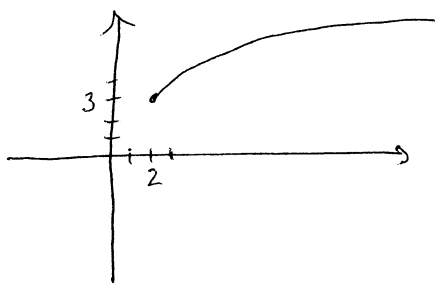
$y = \sqrt{x + 3}$

Shift left by 3

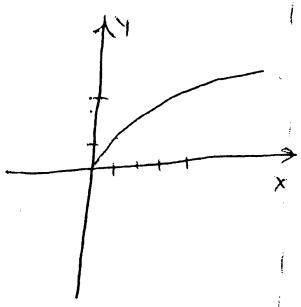


$y = \sqrt{x - 2} + 3$

Shift right by 2, up by 3

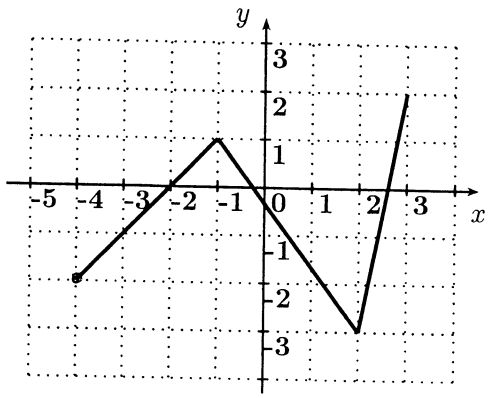


Recall $y = \sqrt{x}$



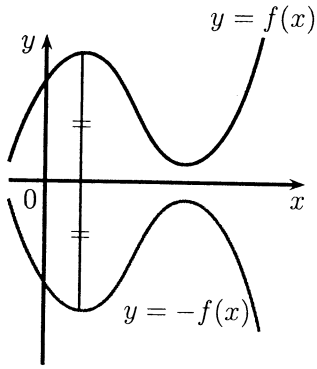
Example 3:

The graph of $y = f(x)$ is shown below.



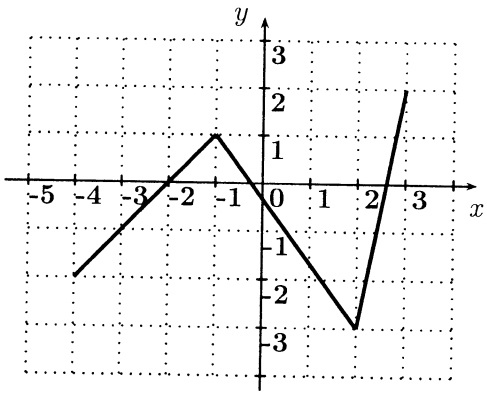
▶ **Reflecting Graphs:**

To graph $y = -f(x)$,
reflect the graph of $y = f(x)$ in the x-axis.



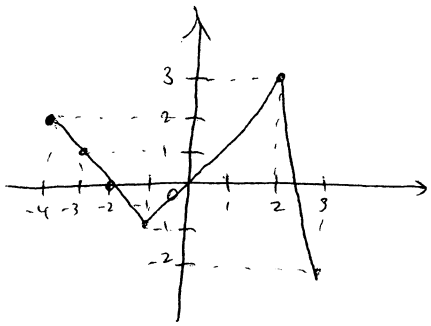
Example 4:

The graph of $y = f(x)$ is shown below.



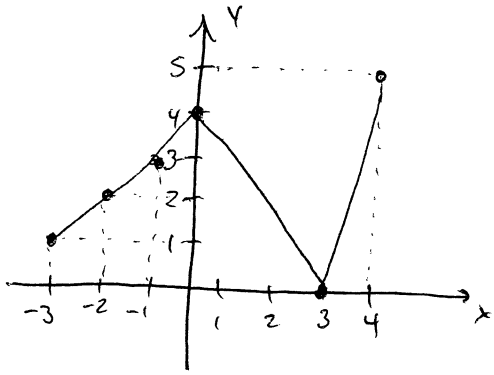
Sketch the graph of $y = -f(x)$.

reflect on x-axis

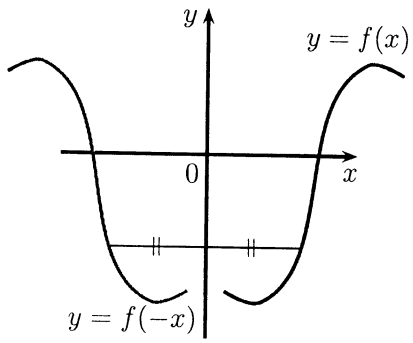


Sketch the graph of $y = f(x-1)+3$.

Shift right by 1, up by 3

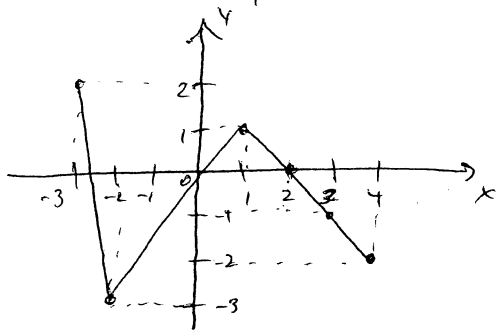


To graph $y = f(-x)$,
reflect the graph of $y = f(x)$ in the y-axis



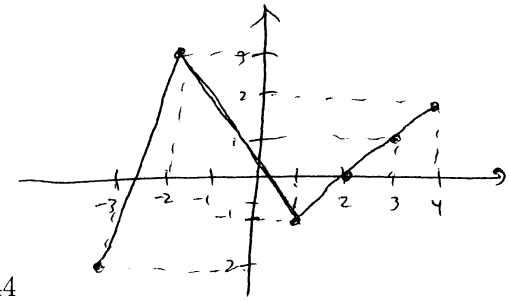
Sketch the graph of $y = f(-x)$.

reflect on y-axis



Sketch the graph of $y = -f(-x)$.

reflect on both axes

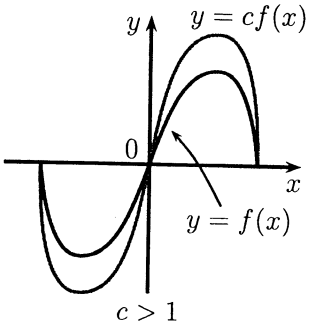


▶ **Vertical Stretching and Shrinking:**

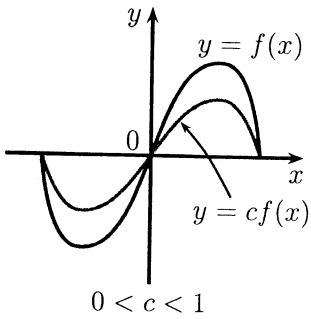
To graph $y = cf(x)$:

If $c > 1$, STRETCH the graph of $y = f(x)$ vertically by a factor of c.

If $0 < c < 1$, SHRINK the graph of $y = f(x)$ vertically by a factor of c.



$c > 1$



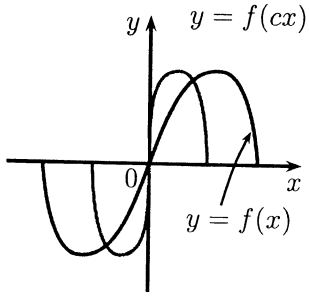
$0 < c < 1$

▶ **Horizontal Shrinking and Stretching:**

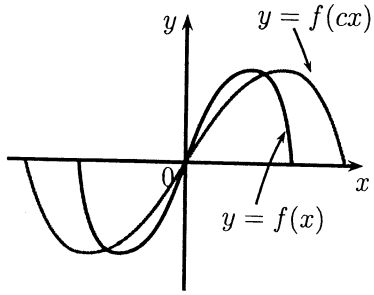
To graph $y = f(cx)$:

If $c > 1$, SHRINK the graph of $y = f(x)$ horizontally by a factor of $1/c$.

If $0 < c < 1$, STRETCH the graph of $y = f(x)$ horizontally by a factor of $1/c$.



$c > 1$

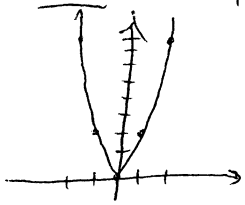


$0 < c < 1$

Example 6: Sketch the graph of:

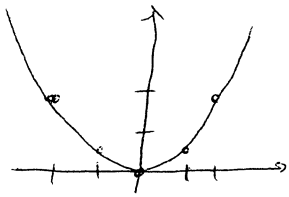
$y = 2x^2$

Stretch vertically by 2

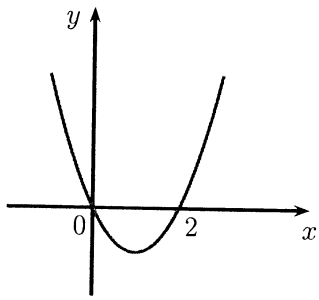


$y = \frac{1}{2}x^2$

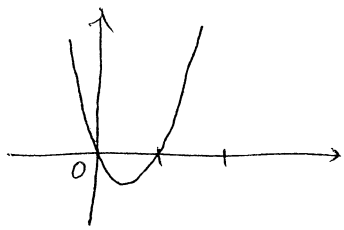
Shrink vertically by $\frac{1}{2}$



Example 7: Use the graph of $f(x) = x^2 - 2x$ provided below to sketch the graph of $f(2x)$.



Shrink horizontally by $\frac{1}{2}$



Example 8:

Use transformations to sketch the graph of

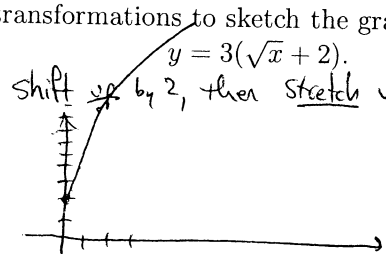
$y = 3\sqrt{x} + 2.$

Stretch vertically by 3, shift up by 2

Use transformations to sketch the graph of

$y = 3(\sqrt{x} + 2).$

Shift up by 2, then stretch vertically by 3



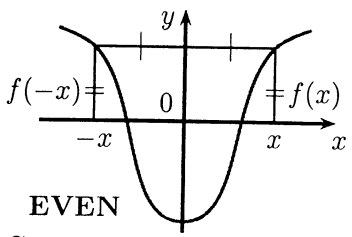
What transformations did you use? Can the transformations be done in any order, or do you have to apply them in a specific order?

You must transform in a specific order. Follow the usual rules of precedance.

▶ **Even and Odd Functions:** Let f be a function.

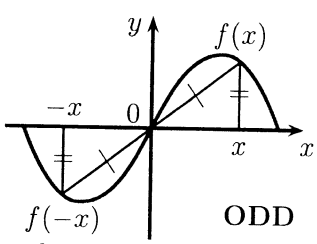
f is **even** if $f(-x) = f(x)$ for all x in the domain of f.

f is **odd** if $f(-x) = -f(x)$ for all x in the domain of f.



EVEN

Graph symmetric wrt y-axis.



ODD

Graph symmetric wrt $(0, 0)$.

Example 9: Determine whether the following functions are even or odd:

$f(x) = x^3 + 2x^5$

$f(-x) = (-x)^3 + 2(-x)^5$

$= -x^3 - 2x^5$

$= -(x^3 + 2x^5)$

$= -f(x)$

⇓

odd function

$g(x) = x^2 - 3x^4$

$g(-x) = (-x)^2 - 3(-x)^4$

$= x^2 - 3x^4$

$= g(x)$

⇓

even function