MA109, Activity 2: Modeling with Equations (Section 1.2, pp. 83-91)   Date: ________

Today's Goal: We practice how to set up (and then solve) equations that model possible problems arising from real-world situations.

Assignments: **Homework** (Sec. 1.2): # 2, 11, 13, 19, 21, 24, 26, 33, 35, 36, 42, 43, 47 (pp. 92-95).

Many problems in the sciences, economics, finance, medicine, and numerous other fields can be formulated into algebraic terms: That's one reason why Algebra is so important.

Here are some guidelines to help you set up equations that model situations described in words.

Guidelines for Modeling with Equations

- Identify the Variable
- Express all Unknown Quantities in Terms of the Variable
- Set Up the Model
- Solve the Equation and Check Your Answer

Example 1 (Investments): Phyllis invested \$12,000, a portion earning a simple interest rate of $4\frac{1}{2}\%$ per year and the rest earning a rate of 4% per year. After one year the total interest earned on these investments was \$525. How much did she invest at each rate?

x at rate 4.5 %

y at rate 4 %

$x+y = 12{,}000 \Rightarrow y = 12000 - x$

$$\frac{4.5}{100}x + \frac{4}{100}(12000 - x) = 525$$

$$\frac{4.5x}{100} + 480 - \frac{4x}{100} = 525$$

$$\frac{.5x}{100} = 45$$

$$.5x = 4500$$

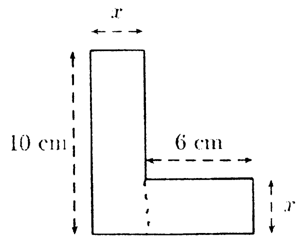
$$x = 9000$$

$$y = 12000 - 9000 = 3000$$

She invest \$9000 at rate 4.5%

\$3000 at rate 4%

Example 2 (Geometry):
Find the length x in the figure, if the total area is 144 cm^2.
(**Note:** The figure is not drawn to scale.)



$$10x + 6x = 144$$

$$16x = 144$$

$$x = \frac{144}{16}$$

Therefore $x = 9$ cm

Example 3 (Mixture Problem): The radiator in a car is filled with a solution of 60% antifreeze and 40% water. The manufacturer of the antifreeze suggests that, for summer driving, optimal cooling of the engine is obtained with only 50% antifreeze. If the capacity of the radiator is 3.6 liters, how much coolant should be drained and replaced with water to reduce the antifreeze concentration to the recommended level?

$\frac{50}{100} \cdot 3.6 = 1.8$ liters

$x =$ the amount of solution to be drain

After draining, there is $3.6 - x$ coolant

$\frac{60}{100}(3.6 - x)$ is antifreeze

After replacing by x liter of water the amount of anti freeze still $.6(3.6 - x)$

$$.6(3.6 - x) = 1.8$$
$$3.6 - x = 3$$
$$x = 3.6 - 3 = .6\ \ell$$

Need to drain $.6\ \ell$ of coolant.

Example 4 (Distance, Speed, and Time): Two cyclists, 90 miles apart, start riding toward each other at the same time. One cycles twice as fast as the other. If they meet 2 hours later, at what average speed is each cyclist traveling?

$x =$ the speed of the 1st cyclist

$2x =$ the speed of the other cyclist

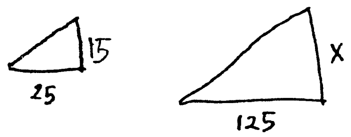
After 2 hours

distance traveled by 1st cyclist + distance traveled by 2nd cyclist = 90

$$2x + 2(2x) = 90$$
$$6x = 90$$
$$x = 15$$

Therefore the speed of the cyclists are 15 mph and 30 mph

Example 5 (Height of a Tree): A woodcutter determines the height of a tall tree by first measuring a smaller one 125 ft away, then moving so that his eyes are in the line of sight along the tops of the trees, and measuring how far he is standing from the small tree. Suppose the small tree is 20 ft tall, the man is 25 ft from the small tree, and his eye level is 5 ft above the ground. How tall is the taller tree?

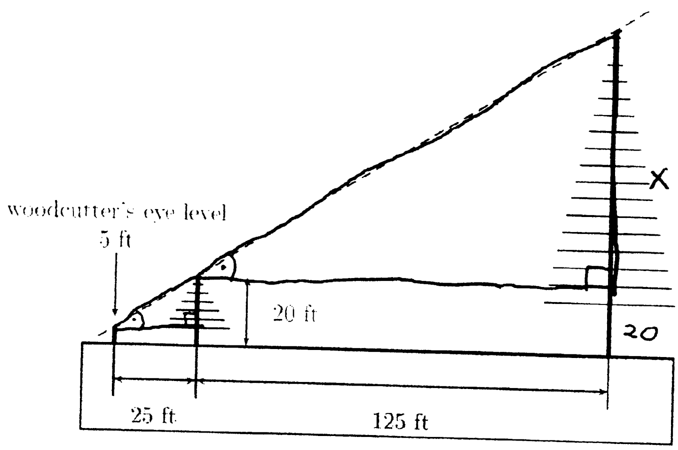


$$\frac{x}{15} = \frac{125}{25}$$

$$\frac{x}{15} = 5$$

$$x = 75$$

Therefore the taller tree has height $75 + 20 = 95$ ft.



(**Note:** This problem is more challenging than the previous ones. The picture in the text on page 94 might be more helpful, even though out of scale.)