MA109, Activity 4: Other Types of Equations (Section 1.5, pp. 115-121) Date: ________

Today's Goal: So far we have learned how to solve linear and quadratic equations. We now study other types of equations, including those that involve higher powers, fractional expressions, and radicals.

Assignments: **Homework (Sec. 1.5):** # 1, 2, 7, 13, 21, 23, 25, 33, 40, 43, 49, 67, 69, 72 (pp. 122-124).

▶ Polynomial Equations:

Some equations can be solved by factoring and using the Zero-Product Property, which says that if a product is zero, then at least one of the factors must equal 0.

Example 1: Find all real solutions of the equation $x^5 = 27x^2$

$x^5 - 27x^2 = 0$

$x^2(x^3 - 27) = 0$

$x^2 = 0$ or $(x^3 - 27) = 0$

$\boxed{x = 0 \quad \text{or} \quad x = \sqrt[3]{27}}$

Example 2: Find all real solutions of the equation $7x^3 - x + 1 = x^3 + 3x^2 + x$

$6x^3 - 3x^2 - 2x + 1 = 0$

$3x^2(2x - 1) - 1(2x - 1) = 0$

$(3x^2 - 1)(2x - 1) = 0$

$(3x^2 - 1) = 0$ or $(2x - 1) = 0$

$x^2 = 1/3$ or $x = 1/2$

$\boxed{x = \pm\sqrt{1/3} \quad \text{or} \quad x = 1/2}$

▶ Equations Involving Radicals:

When we solve an equation, we may end up with one or more **extraneous solutions**, that is solutions that do not satisfy the original equation. That is why you **must** always check your answers to make sure that each solution satisfies the original equation

Example 3: Find all real solutions of the equation $\sqrt{x+4} = x + 2$

$x + 4 = (x + 2)^2$

$x + 4 = x^2 + 4x + 4$

$0 = x^2 + 3x$

$0 = x(x + 3)$

$x = 0$ or $x = -3$

Check Solutions

check 0: $\sqrt{0+4} = 0 + 2$

$2 = 2$ ✓

check -3: $\sqrt{-3+4} = -3 + 2$

$\sqrt{1} = -1$

$1 \neq -1$ X

Solution

$\boxed{x = 0}$

Example 4: Find all real solutions of the equation $\sqrt{x + \sqrt{x+2}} = 2$

$x + \sqrt{x+2} = 4$

$\sqrt{x+2} = 4 - x$

$x+2 = (4-x)^2$

$x+2 = 16 - 8x + x^2$

$0 = x^2 - 9x + 14$

$0 = (x-7)(x-2)$

$x = 7$ or $x = 2$

Check Solutions

check $x=2$: $\sqrt{2+\sqrt{2+2}} = 2$

$\sqrt{2+2} = 2$

$\sqrt{4} = 2$ ✓

check $x=7$: $\sqrt{7+\sqrt{7+2}} = 2$

$\sqrt{7+3} = 2$

$\sqrt{10} \neq 2$ X

Answer: $\boxed{x = 2}$

▶ **Equations of Quadratic Type:**

An equation such as $au^2 + bu + c = 0$, where u is an algebraic expression, is an equation of **quadratic type**.

Example 5: Solve the equation $x^4 - 5x^2 + 4 = 0$

$u = x^2$

$u^2 = (x^2)^2 = x^4$

$x^4 - 5x^2 + 4 = 0$

becomes

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4$ or $u = 1$

$u = 4$

$u = x^2$

$4 = x^2$

$\pm\sqrt{4} = x$

$\boxed{\pm 2 = x}$

or

$u = 1$

$u = x^2$

$1 = x^2$

$\pm\sqrt{1} = x$

$\boxed{\pm 1 = x}$

Example 6: Find all real solutions of the equation $x^{1/3} + x^{1/6} - 2 = 0$

$u = x^{1/6}$

$u^2 = (x^{1/6})^2$

$x^{1/3} + x^{1/6} - 2 = 0$

becomes

$u + u^2 - 2 = 0$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2$ or $u = 1$

$u = -2$

$u = x^{1/6}$

$-2 = x^{1/6}$

$-2 = \sqrt[6]{x}$

No sol'n

X

$u = 1$

$u = x^{1/6}$

$1 = x^{1/6}$

$1^6 = x$

$\boxed{1 = x}$

[redacted]

Example 7: Find all real solutions of the equation $\left(\dfrac{x}{x+2}\right)^2 = \dfrac{4x}{x+2} - 4$

$u = \left(\frac{x}{x+2}\right)$

$u^2 = \left(\frac{x}{x+2}\right)^2$

$\left(\frac{x}{x+2}\right)^2 = \frac{4x}{x+2} - 4$

becomes

$u^2 = 4u - 4$

$u^2 - 4u + 4 = 0$

$(u-2)(u-2) = 0$

$u = 2$

$\underline{u=2}$

$u = \left(\frac{x}{x+2}\right)$

$2 = \frac{x}{x+2}$

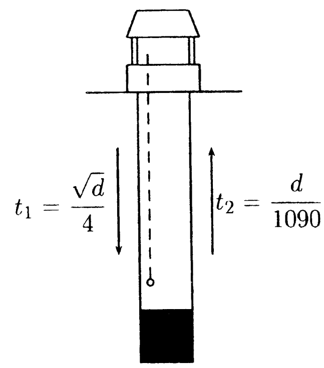
$2(x+2) = x$

$2x + 4 = x$

$\boxed{x = -4}$

▶ **Applications:** Many real-life problems can be modeled with the types of equations we have just studied.

Example 8:
One method for determining the depth of a well is to drop a stone into it and then measure the time it takes until the splash is heard. If d is the depth of the well (in feet) and t_1 the time (in seconds) it takes for the stone to fall, then $d = 16t_1^2$, so $t_1 = \sqrt{d}/4$. Now if t_2 is the time it takes for the sound to travel back up, then $d = 1090t_2$ because the speed of sound is 1090 feet per second. So $t_2 = d/1090$. Thus the total time elapsed between dropping the stone and hearing the splash is $t_{\text{tot}} = t_1 + t_2 = \dfrac{\sqrt{d}}{4} + \dfrac{d}{1090}$.
How deep is the well if this total time is 3 seconds?



$3 = \frac{\sqrt{d}}{4} + \frac{d}{1090}$

$u = \sqrt{d}$

$u^2 = d$

$3 = \frac{\sqrt{d}}{4} + \frac{d}{1090}$

becomes

$3 = \frac{u}{4} + \frac{u^2}{1090}$

$12 = u + \frac{4u^2}{1090}$

$13080 = 1090u + 4u^2$

$0 = 4u^2 + 1090u - 13080$

$0 = 2u^2 + 545u - 6540$

$u_1, u_2 = \frac{-545 \pm \sqrt{(545)^2 - 4(2)(-6540)}}{2(2)}$

$u_1, u_2 = \frac{-545 \pm \sqrt{349345}}{4}$

$u_1 = \frac{-545 + \sqrt{349345}}{4}$

$u_1 \approx 11.51$

$\sqrt{d} \approx 11.51$

$d \approx 132.56\text{ ft}$

$u_2 = \frac{-545 - \sqrt{349345}}{4}$

$u_2 \approx -284.01$

$\sqrt{d} \approx -284.01$

No sol'n

[redacted]