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MA109, Activity 5: Inequalities(Section 1.6, pp. 124-132)

Today's Goal:	Some problems in algebra lead to 'inequalities' instead of equations. We learn how to 'solve' inequalities.
Assignments:	**Homework** (Sec. 1.6): # 9, 14, 17, 23, 26, 31, 33, 44, 48, 51, 56, 73, 79 (pp. 132-134).

Some problems in algebra lead to **inequalities** instead of equations. For instance: $3x - 1 \geq 3 + x$.

To **solve** an inequality that contains a variable means to find all values of the variable that make the inequality true. Unlike an equation, an inequality generally has infinitely many solutions, which form an interval or a union of intervals on the real line.

To solve inequalities, we use the following rules to isolate the variable on one side of the inequality sign. In these rules the symbols A, B, C and D stand for real number or algebraic expressions.

Note: All of these rules also apply to the other inequality symbols ($A \geq B$, $A < B$, $A > B$).

Rules for Inequalities:

1. $A \leq B \Leftrightarrow A + C \leq B + C$
2. $A \leq B \Leftrightarrow A - C \leq B - C$
3. If $C > 0$, then $A \leq B \Leftrightarrow AC \leq BC$
4. If $C < 0$, then $A \leq B \Leftrightarrow AC \geq BC$
5. If $A > 0$ and $B > 0$, then $A \leq B \Leftrightarrow \frac{1}{A} \geq \frac{1}{B}$
6. If $A \leq B$ and $C \leq D$, then $A + C \leq B + D$

▶ **Linear Inequalities:** An inequality is **linear** if each term is constant or a multiple of the variable.

Example 1: Solve each inequality. Express the solution using interval notation and graph the solution set.

- $3x - 1 \geq 3 + x$

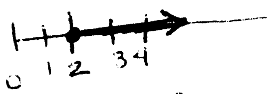
$3x - 1 \geq 3 + x$

$-x \qquad -x$

$2x - 1 \geq 3$

$+1 \quad +1$

$\frac{2x}{2} \geq \frac{4}{2} \Rightarrow x \geq 2$



$[2, \infty)$

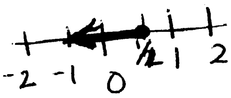
- $6 - 2(1 - x) \leq 5$

$6 - 2 + 2x \leq 5$

$4 + 2x \leq 5$

$-4 \qquad -4$

$\frac{2x}{2} \leq \frac{1}{2} \Rightarrow x \leq \frac{1}{2}$



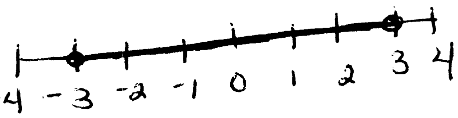
$(-\infty, \frac{1}{2}]$

- $-4 < 2 - 2x < 8$

$-2 \quad -2 \qquad -2$

$\frac{-6}{-2} < \frac{-2x}{-2} < \frac{6}{-2}$

$3 > x > -3$



$[-3, 3]$

▶ **Nonlinear Inequalities:** To solve inequalities involving squares and other powers of the variable, we use factoring together with the following principle:

The Sign of a Product or Quotient

1. If a product or a quotient has an EVEN number of negative factors, then the whole expression is positive.
2. If a product or a quotient has an ODD number of negative factors, then the whole expression is negative.

Example 2: Solve the inequality $x^2 + x - 12 \le 0$

$(x+4)(x-3) \le 0$

Critical values: $-4, 3$

Test values: $-5, 0, 4$

Sign of $(x+4)(x-3)$: + + + +   − − − −   + + + +

$-5 \quad -4 \quad 0 \quad 3 \quad 4$

$(-\infty, -4]$ $[-4, 3]$ $[3, \infty)$

Solution: $[-4, 3]$

Guidelines for Solving Nonlinear Inequalities

1. Move all terms to one side. If the non-zero side of the inequality involves quotients, bring them to a common denominator.
2. Factor the non-zero side of the inequality
3. Find the intervals by determining the values for which each factor is zero. These numbers will divide the real line into intervals.
4. Make a table or diagram and test a value within each interval to determine the sign of the expression on that interval.

Example 3: Express the solution using interval notation and graph the solution set.

- $x^5 > x^3$

$-x^3 \quad -x^3$

$x^5 - x^3 > 0$

$x^3(x^2-1) > 0$

$x^3(x-1)(x+1) > 0$

Critical Values: $x = 0, 1, -1$

− − −   + + +   − − −   + + +

$-2 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad 2$

Test Values: $-2, -\frac{1}{2}, \frac{1}{2}, 2$

Solution: $(-1, 0) \cup (1, \infty)$

- $\dfrac{2+x}{2-x} \ge 1$

$\dfrac{2+x}{2-x} - 1 \ge 0$

$\dfrac{2+x}{2-x} - \dfrac{2-x}{2-x} \ge 0$

$\dfrac{2+x-(2-x)}{2-x} \ge 0$

$\dfrac{2+x-2+x}{2-x} \ge 0$

$\dfrac{2x}{2-x} \ge 0$

Critical #'s

$2x = 0 \qquad 2-x=0$

$x = 0 \qquad 2 = x$

$0 \quad 2$

Test Values: $-1, 1, 3$

Give us:

−   +   −

$0 \quad 2$

Since $2-x$ is in denom., $x \ne 2$

Solution: $[0, 2)$

[redacted]

Critical Values: $x=0$, $3x+2=0 \Rightarrow x=-2/3$

$x+1=0 \Rightarrow x=-1$

$0, -2/3, -1$

- $3x < \dfrac{x}{x+1}$

$-\dfrac{x}{x+1} \quad -\dfrac{x}{x+1}$

$3x - \dfrac{x}{x+1} < 0$

$\dfrac{3x(x+1)}{x+1} - \dfrac{x}{x+1} < 0 \Rightarrow \dfrac{3x^2+3x-x}{x+1} < 0$

$\dfrac{3x^2+2x}{x+1} < 0 \Rightarrow \dfrac{x(3x+2)}{x+1} < 0$

$- - - \quad + + + \quad - - - \quad + + +$

$-2 \quad -1 \quad -2/3 \quad 0 \quad 1$

Test Values: $-2, -5/6, -1/2, 1/2, 1$

Solution: $(-\infty, -1) \cup (-2/3, 0)$

▶ **Modeling with Inequalities:** Modeling real-life problems frequently leads to inequalities because we are often interested in determining when one quantity is more (or less) than another.

Example 4 (Airline Ticket Price): A charter airline finds that on its Saturday flights from Philadelphia to London, all 120 seats will be sold if the ticket price is \$200. However, for each \$3 increase in ticket price, the number of seats sold decreases by one.

(a) Find a formula for the number of seats sold if the ticket price is P dollars.

Seats $= -\frac{1}{3}P + \dfrac{560}{3}$

Set up system:

$-120 = -200a - b$

$110 = 230a + b$

$-10 = 30a$

$a = -1/3$

$120 = 200(-1/3) + b$

$120 = -\dfrac{200}{3} + b$

$\dfrac{360+200}{3} = b \Rightarrow b = \dfrac{560}{3}$

(b) Over a certain period, the number of seats sold for this flight ranged between 90 and 115. What was the corresponding range of ticket prices?

$3\left(90 < -\frac{1}{3}P + \frac{560}{3} < 115\right)$

$270 < -P + 560 < 345$

$-560 \quad -560 \quad -560$

$-1(-290 < -P < 215) \Rightarrow 290 > P > 215$

Range of P: \$215 – \$290

Example 5 (Gas Mileage): The gas mileage g (measure in mi/gal) for a particular vehicle, driven at v mi/h, is given by the formula $g = 10 + 0.9v - 0.01v^2$, as long as v is between 10 mi/h and 75 mi/h. For what range of speed is the vehicle's mileage 30 mi/gal or better?

$30 \leq 10 + .09v - .01v^2$

$0 \leq -20 + .09v - .01v^2 \Rightarrow -1(-.01v^2 + .09v - 20 \geq 0)$

$.01v^2 - .09v + 20 \geq 0$

Quadratic Formula

$\dfrac{.09 \pm \sqrt{(-.09)^2 - 4(.01)(20)}}{2(.01)} = \dfrac{.9 \pm \sqrt{.81 - .80}}{.02} \Rightarrow \dfrac{.9 \pm .1}{.02}$

$1/.02 = 50$

$.8/.02 = 40$

Range of Speed: 40mph – 50mph