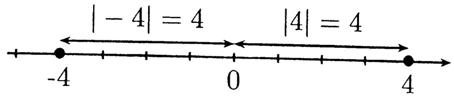
MA109, Activity 6: Absolute Value Equations and Inequalities **(Section 1.7, pp. 135-137)** **Date:** __________

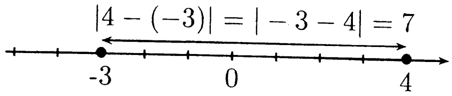
Today's Goal:	We learn how to solve equations and inequalities that involve an absolute value.
Assignments:	**Homework (Sec. 1.7):** #1, 5, 8, 11, 17, 19, 24, 31, 37, 43, 47, 51 (pp. 137-138).

The absolute value of a number a is given by

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$



and it represents the distance from a to the origin on the real number line. In general, $|b - a|$ is the distance between b and a on the real number line.



▶ **Absolute Value Equations:**

We use the following property to solve equations that involve an absolute value:

$$\boxed{|x| = C \quad \Leftrightarrow \quad x = \pm C}$$

This means that to solve an absolute value equation, we must solve *two* separate equations.

Example 1: Solve the equation $|x + 4| = 0.5$

$|x+4| = 0.5 \Leftrightarrow x+4 = \pm 0.5$

$x+4 = 0.5$ or $x+4 = -0.5$

$x = 0.5-4$ $x = -0.5-4$

$\boxed{x = -3.5}$ $\boxed{x = -4.5}$

Ck: $|-3.5+4| \stackrel{?}{=} 0.5$ $|-4.5+4| \stackrel{?}{=} 0.5$ (It is a good idea to check your answers to make sure you didn't make a mistake.)

$|0.5| \stackrel{?}{=} 0.5$ $|-0.5| \stackrel{?}{=} 0.5$

$0.5 = 0.5$ ✓ $0.5 = 0.5$ ✓

Example 2: Solve the equation $3|x + 5| + 6 = 15$

We need the absolute value on one side by itself, so we can solve for it.

$3|x+5|+6 = 15$

$3|x+5| = 15-6$

$3|x+5| = 9$

$|x+5| = 3 \Leftrightarrow x+5 = \pm 3$

$x+5 = 3$ or $x+5 = -3$

$x = 3-5$ $x = -3-5$

$\boxed{x = -2}$ $\boxed{x = -8}$

Ck: $3|-2+5|+6 \stackrel{?}{=} 15$ $3|-8+5|+6 \stackrel{?}{=} 15$

$3|3|+6 \stackrel{?}{=} 15$ $3|-3|+6 \stackrel{?}{=} 15$

$3(3)+6 \stackrel{?}{=} 15$ $3(3)+6 \stackrel{?}{=} 15$

$9+6 \stackrel{?}{=} 15$ $9+6 \stackrel{?}{=} 15$

$15 = 15$ ✓ $15 = 15$ ✓

Example 3: Solve the equation $|x+3| = |2x+1|$

We can just put the $\pm$ on one side since putting it on both sides yields duplicate equations.

$|x+3| = |2x+1| \Rightarrow x+3 = \pm(2x+1)$

$x+3 = 2x+1$
$x+3-2x = 1$
$-x+3 = 1$
$-x = 1-3$
$-x = -2$
$x = 2$

$x+3 = -(2x+1)$
$x+3 = -2x-1$
$x+3+2x = -1$
$3x+3 = -1$
$3x = -1-3$
$3x = -4 \Rightarrow x = -\frac{4}{3}$

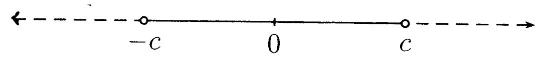
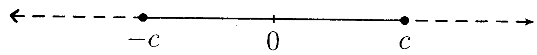
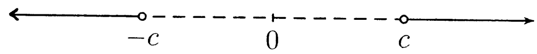
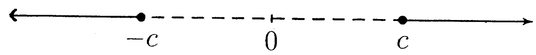
Ck: $x=2$ $\quad |2+3| \stackrel{?}{=} |2(2)+1|$
$|5| \stackrel{?}{=} |4+1|$
$|5| \stackrel{?}{=} |5|$
$5 = 5$ ✓

$x = -\frac{4}{3}$ $\quad |-\frac{4}{3}+3| \stackrel{?}{=} |2(-\frac{4}{3})+1|$
$|\frac{5}{3}| \stackrel{?}{=} |-\frac{8}{3}+1|$
$|\frac{5}{3}| \stackrel{?}{=} |-\frac{5}{3}|$
$\frac{5}{3} = \frac{5}{3}$ ✓

► **Absolute Value Inequalities:**

We use the following properties to solve inequalities that involve an absolute value.

Properties of Absolute Value Inequalities:

Inequality	Equivalent form	Graph	Interval notation
1. $\lvert x\rvert < c$	$-c < x < c$		$(-c, c)$
2. $\lvert x\rvert \le c$	$-c \le x \le c$		$[-c, c]$ Notice. This should be a bracket.
3. $\lvert x\rvert > c$	$x < -c$ or $c < x$		$(-\infty, -c) \cup (c, +\infty)$
4. $\lvert x\rvert \ge c$	$x \le -c$ or $c \le x$		$(-\infty, -c] \cup [c, +\infty)$

Example 4:

Solve the inequality $|x| < 5$. Graph the solution set, and express the solution using interval notation.

$|x| < 5 \xrightarrow{\text{Property 1}} -5 < x < 5$

$(-5, 5)$

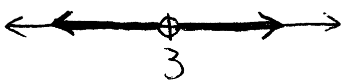
Example 5:

Solve the inequality $|x-3| > 0$. Graph the solution set, and express the solution using interval notation.

$|x-3| > 0 \xrightarrow{\text{Property 3}} x-3 < -(0)$ or $0 < x-3$

$x-3 < 0 \qquad 3 < x$

$x < 3$



$(-\infty, 3) \cup (3, \infty)$

Example 6:

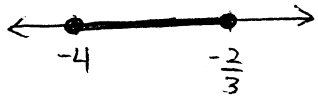
Solve the inequality $|3x+7| \le 5$. Graph the solution set, and express the solution using interval notation.

$$|3x+7| \le 5 \xrightarrow[\text{②}]{\text{Property}} -5 \le 3x+7 \le 5$$

$$-5-7 \le 3x \le 5-7$$

$$-12 \le 3x \le -2$$

$$-4 \le x \le -\frac{2}{3}$$



$$\left[-4, -\frac{2}{3}\right]$$

Example 7:

Solve the inequality $\dfrac{1}{|2x-3|} \le 5$. Graph the solution set, and express the solution using interval notation.

We need to work with our inequality to get it into a form were we can apply a property.

$$\frac{1}{|2x-3|} \le 5$$

$$1 \le 5|2x-3|$$

$$\frac{1}{5} \le |2x-3|$$

So $|2x-3| \ge \frac{1}{5}$.

$$|2x-3| \ge \frac{1}{5} \xrightarrow[\text{④}]{\text{Property}} 2x-3 \le -\frac{1}{5},\ \frac{1}{5} \le 2x-3$$

$$2x-3 \le -\frac{1}{5} \qquad \frac{1}{5} \le 2x-3$$

$$2x \le -\frac{1}{5}+3 \qquad \frac{1}{5}+3 \le 2x$$

$$2x \le \frac{14}{5} \qquad \frac{16}{5} \le 2x$$

$$x \le \frac{14}{5}\left(\frac{1}{2}\right) \qquad \left(\frac{1}{2}\right)\left(\frac{16}{5}\right) \le x$$

$$x \le \frac{7}{5} \qquad \frac{8}{5} \le x$$

We also need to check where the denominator equals 0. This occurs when $|2x-3|=0$, so $2x-3=0$, $2x=3$, $x=\frac{3}{2}$. We need to exclude this from our solutions. (continued below)

Example 8: The average height of adult males is 68.2 inches, and 95% of adult males have height h that satisfies the inequality $\left|\dfrac{h-68.2}{2.9}\right| \le 2$. Solve the inequality to find the range of heights.

$$\left|\frac{h-68.2}{2.9}\right| \le 2 \xrightarrow[\text{②}]{\text{Property}} -2 \le \frac{h-68.2}{2.9} \le 2$$

$$-2(2.9) \le h-68.2 \le 2(2.9)$$

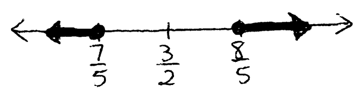
$$-5.8 \le h-68.2 \le 5.8$$

$$-5.8+68.2 \le h \le 5.8+68.2$$

$$\boxed{62.4 \le h \le 74}$$

Example 7 continued

So we have $x \le \frac{7}{5}$, $\frac{8}{5} \le x$, and $x \ne \frac{3}{2}$.



Since $\frac{3}{2}$ was not in our possible solutions, it does not affect our answer.

$$\left(-\infty, \frac{7}{5}\right] \cup \left[\frac{8}{5}, \infty\right)$$