

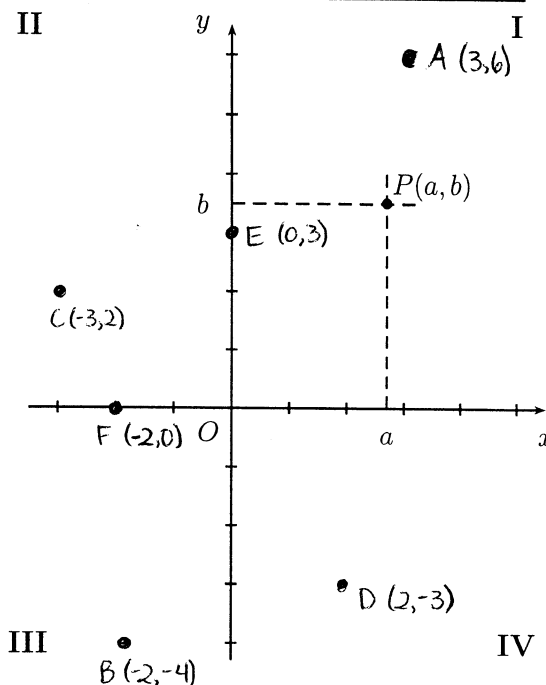
Today's Goal: The coordinate plane is the link between algebra and geometry. In the coordinate plane we can draw graphs of algebraic equations. The graphs, in turn, allow us to 'see' the relationship between the variables in equations.

Assignments: **Homework (Sec. 2.1):** #1, 2, 5, 8, 9, 14, 16, 19, 25, 28, 33, 39, 43 (pp. 154-157).

Points in a plane can be identified with ordered pairs of numbers to form the *coordinate plane*. To do this, we draw two perpendicular oriented lines (one horizontal and the other vertical) that intersect at 0 on each line. The horizontal line with positive direction to the right is called the *x-axis*; the other line with positive direction upward is called the *y-axis*. The point of intersection of the two axes is the *origin* O . The two axes divide the plane into four *quadrants*, labeled I, II, III, and IV.

The coordinate plane is also called *Cartesian plane* in honor of the French mathematician/philosopher René Descartes (1596-1650).

Any point P in the coordinate plane can be located by a unique ordered pair of numbers (a, b) as shown in the picture. The first number a is called the *x-coordinate* of P ; the second number b is called the *y-coordinate* of P .



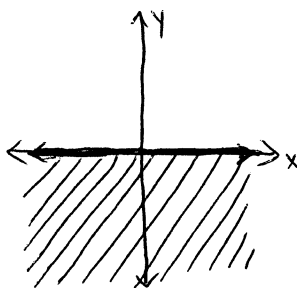
Example 1:

Plot each point in the coordinate plane:
 $A(3, 6)$, $B(-2, -4)$, $C(-3, 2)$, $D(2, -3)$, $E(0, 3)$, $F(-2, 0)$

Example 2: Describe and sketch the regions in the coordinate plane given by the following sets:

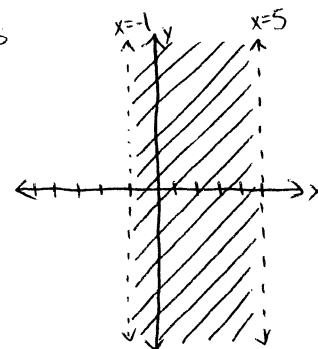
$\{(x, y) \mid y \leq 0\}$

The set of points (x, y) such that $y \leq 0$ is the points whose *y-coordinates* are 0 or negative, i.e. the points on the *x-axis* or below it.



$\{(x, y) \mid -1 < x < 5\}$

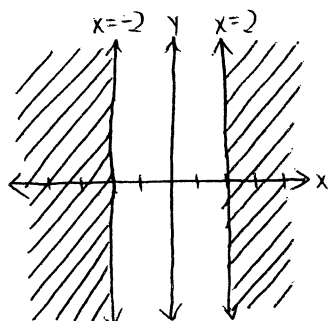
The points whose *x-coordinates* are between -1 and 5 lie between (but not on) the lines $x = -1$ and $x = 5$.



$\{(x, y) \mid |x| \geq 2\}$

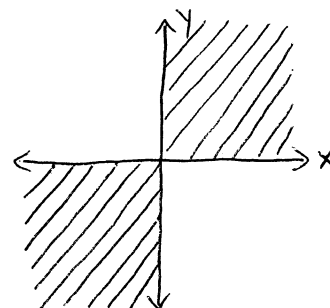
Recall $|x| \geq 2$ if and only if $x \leq -2$, $2 \leq x$.

So the given region consists of the points whose *x-coordinates* lie to the left of or on $x = -2$ and to the right of or on $x = 2$.



$\{(x, y) \mid xy > 0\}$

For $xy > 0$, then x and y must both be positive or both be negative. So we get all points in quadrants I and III.



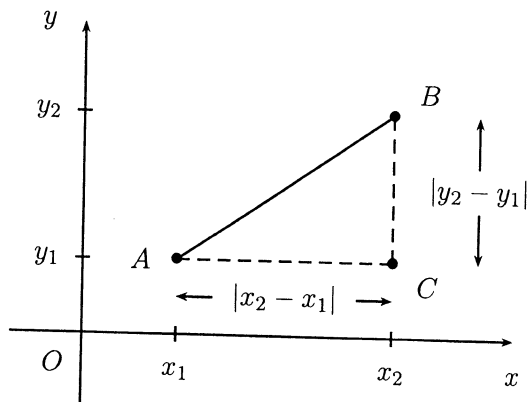
► **The Distance Formula:**

Recall from Activity 2 (Section P.2) that the distance between points a and b on a number line is $d(a, b) = |b - a|$.

Thus, from the picture we see that the distance between the points $A(x_1, y_1)$ and $C(x_2, y_1)$ on a horizontal line is $|x_2 - x_1|$ and the distance between the points $B(x_2, y_2)$ and $C(x_2, y_1)$ on a vertical line is $|y_2 - y_1|$.

Since the triangle ABC is a right triangle, the Pythagorean Theorem gives:

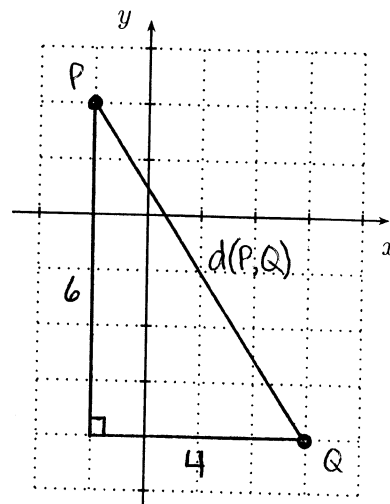
$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 3: Find the distance between $P(-1, 2)$ and $Q(3, -4)$

- Ⓐ • by drawing a right triangle and using the Pythagorean Theorem;
- Ⓑ • by using the distance formula.

$$\begin{aligned} \text{Ⓐ } [d(P, Q)]^2 &= 6^2 + 4^2 & \text{Ⓑ } d(P, Q) &= \sqrt{(3 - (-1))^2 + (-4 - 2)^2} \\ [d(P, Q)]^2 &= 36 + 16 & &= \sqrt{4^2 + (-6)^2} \\ [d(P, Q)]^2 &= 52 & &= \sqrt{16 + 36} \\ d(P, Q) &= \sqrt{52} & &= \sqrt{52} \end{aligned}$$



Example 4: Show that the triangle with vertices $A(0, 2)$, $B(-3, -1)$, and $C(-4, 3)$ is isosceles.

A triangle is isosceles if two of its sides are of equal length, i.e. the distance between two sets of vertices is equal. So we need to find $d(A, B)$, $d(A, C)$, and $d(B, C)$.

$$d(A, B) = \sqrt{(-3 - 0)^2 + (-1 - 2)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18}$$

$$d(A, C) = \sqrt{(-4 - 0)^2 + (3 - 2)^2} = \sqrt{(-4)^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$d(B, C) = \sqrt{(-4 - (-3))^2 + (3 - (-1))^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{1 + 16} = \sqrt{17}$$

We see $d(A, C) = d(B, C)$
so the triangle is isosceles.

► **The Midpoint Formula:**

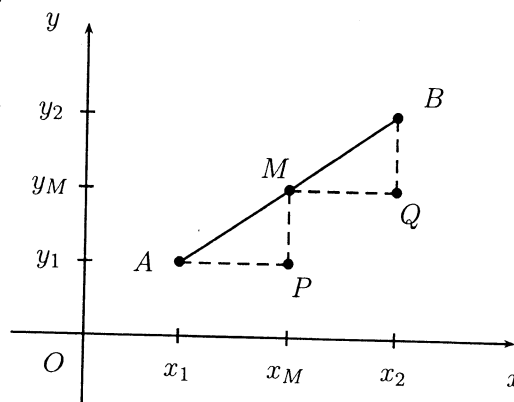
Let (x_M, y_M) denote the coordinates of the midpoint M of the line segment that joins the point $A(x_1, y_1)$ to the point $B(x_2, y_2)$.

From the picture, we notice that the triangles APM and MQB are congruent because $d(A, M) = d(M, B)$ and the corresponding angles are equal.

Thus $d(A, P) = d(M, Q)$ and so $x_M - x_1 = x_2 - x_M$ or $x_M = (x_1 + x_2)/2$.

Similarly, $d(M, P) = d(B, Q)$ from which it follows that $y_M = (y_1 + y_2)/2$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Example 5: If $M(6,8)$ is the midpoint of the line segment AB , and if A has coordinates $(2,3)$, find the coordinates of B .

We have $M = (6,8)$ and we know $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$.

Since $M = (6,8)$ and $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, we have $6 = \frac{x_1+x_2}{2}$ and $8 = \frac{y_1+y_2}{2}$.

We know $(x_1, y_1) = (2,3)$. Plugging these in, we can solve for x_2 and y_2 .

$$6 = \frac{x_1+x_2}{2}$$

$$8 = \frac{y_1+y_2}{2}$$

$$6 = \frac{2+x_2}{2}$$

$$8 = \frac{3+y_2}{2}$$

$$2(6) = 2+x_2$$

$$2(8) = 3+y_2$$

$$12 = 2+x_2$$

$$16 = 3+y_2$$

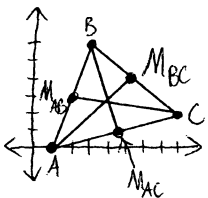
$$12-2 = x_2 \Rightarrow \boxed{x_2 = 10}$$

$$16-3 = y_2 \Rightarrow \boxed{y_2 = 13}$$

So $B = (10,13)$.

Example 6: Find the lengths of the medians of the triangle with vertices $A(1,0)$, $B(3,6)$ and $C(8,2)$. (A median is a line segment from a vertex to the midpoint of the opposite side).

We can first plot the vertices on the coordinate plane. Let M_{AB} denote the midpoint of AB , M_{AC} the midpoint of AC , and M_{BC} the midpoint of BC . We can then draw the medians. In order to find the length of the medians, we must first find the coordinates of the midpoints.



$$M_{AB} = \left(\frac{1+3}{2}, \frac{0+6}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2,3)$$

$$M_{AC} = \left(\frac{1+8}{2}, \frac{0+2}{2}\right) = \left(\frac{9}{2}, \frac{2}{2}\right) = \left(\frac{9}{2}, 1\right)$$

$$M_{BC} = \left(\frac{3+8}{2}, \frac{6+2}{2}\right) = \left(\frac{11}{2}, \frac{8}{2}\right) = \left(\frac{11}{2}, 4\right)$$

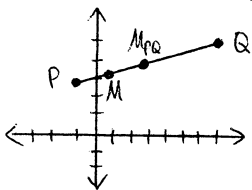
$$d(A, M_{BC}) = \sqrt{\left(\frac{11}{2}-1\right)^2 + (4-0)^2} = \sqrt{\left(\frac{9}{2}\right)^2 + 4^2} = \sqrt{\frac{81}{4} + 16} = \sqrt{\frac{145}{4}}$$

$$d(B, M_{AC}) = \sqrt{\left(\frac{9}{2}-3\right)^2 + (1-6)^2} = \sqrt{\left(\frac{3}{2}\right)^2 + (-5)^2} = \sqrt{\frac{9}{4} + 25} = \sqrt{\frac{109}{4}}$$

$$d(C, M_{AB}) = \sqrt{(2-8)^2 + (3-2)^2} = \sqrt{(-6)^2 + 1^2} = \sqrt{36+1} = \sqrt{37}$$

These are the lengths of our medians.

Example 7: Find the point that is one-fourth of the distance from the point $P(-1,3)$ to the point $Q(7,5)$ along the segment PQ .



To find this point, we can first find the midpoint of PQ , say M_{PQ} and then find the midpoint of P/M_{PQ} , say M .

$$M_{PQ} = \left(\frac{-1+7}{2}, \frac{3+5}{2}\right) = \left(\frac{6}{2}, \frac{8}{2}\right) = (3,4)$$

$$M = \left(\frac{-1+3}{2}, \frac{3+4}{2}\right) = \left(\frac{2}{2}, \frac{7}{2}\right) = \left(1, \frac{7}{2}\right)$$

So our answer is $\left(1, \frac{7}{2}\right)$.