MA109, Activity 8: Graphs of Equations (Section 2.2, pp. 160-166) in Two Variables — Date: ____________

Today's Goal:	We begin learning how to draw the graph of an equation. The graph allows us to see the relationship between the variables in the equation.
Assignments:	**Homework (Sec. 2.2):** # 1,6,7,9,11,12,16,22,25,37,40,42,45,55,58,59,65,69 (pp. 167-170).

An **equation in two variables**, say x and y, expresses a relationship between two quantities.

A point (x_0, y_0) **satisfies** the equation if it makes the equation true when the values x_0 and y_0 are substituted into the equation in place of x and y. For instance, the point $P(3, 7)$ satisfies the equation $y = x^2 - 2$.

The **graph of an equation** in x and y is the set of all points (x_0, y_0) in the coordinate plane that satisfy the equation.

▶ **Graphing Equations by Plotting Points:**

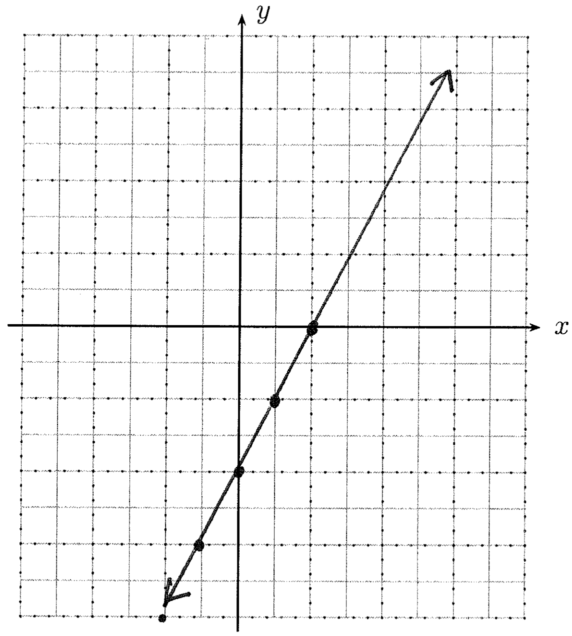
The graph of an equation is a curve, so to graph an equation we plot as many points as we can, then connect them by a smooth curve. In **Calculus** you will learn more sophisticated graphing techniques.

Example 1:

Sketch the graph of the equation $2x - y = 4$

$$-y = 4 - 2x$$
$$y = 2x - 4$$

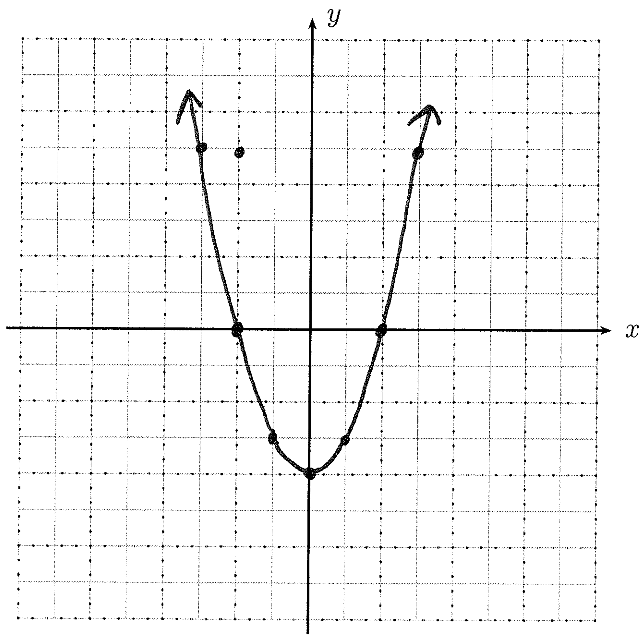
x	y
-2	-8
-1	-6
0	-4
1	-2
2	0



Example 2:

Sketch the graph of the equation $y = x^2 - 4$

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5



▶ **Intercepts:**

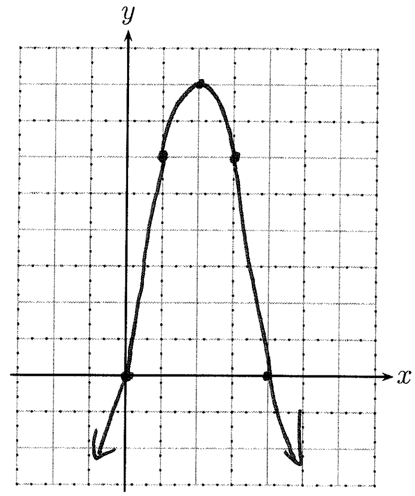
The x-coordinates of the points where a graph intersects the x-axis are called the **x-intercepts** of the graph and are obtained by setting $y = 0$ in the equation of the graph.

The y-coordinates of the points where a graph intersects the y-axis are called the **y-intercepts** of the graph and are obtained by setting $x = 0$ in the equation of the graph.

Example 3: Find the x- and y-intercepts of the graph of the equation

$$y = 8x - 2x^2.$$

Also, sketch the graph of the equation.



x-intercept

Let $y = 0$

$0 = 8x - 2x^2$

$0 = 2x(4 - x)$

$x = 0$ or $4 - x = 0$

$x = 4$

$(0,0)$ or $(4,0)$

y-intercept

Let $x = 0$

$y = 8(0) - 2(0)^2$

$y = 0$

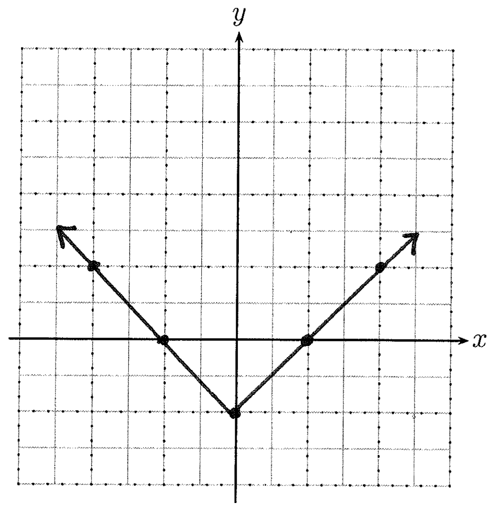
$(0,0)$

x	y
1	6
2	8
3	6

Example 4: Find the x- and y-intercepts of the graph of the equation

$$y = |x| - 2.$$

Also, sketch the graph of the equation.



x-intercept

Let $y = 0$

$0 = |x| - 2$

$2 = |x|$

$x = 2$ or $x = -2$

$(2,0)$✓ or $(-2,0)$✓

y-intercept

Let $x = 0$

$y = |0| - 2$

$y = 0 - 2$

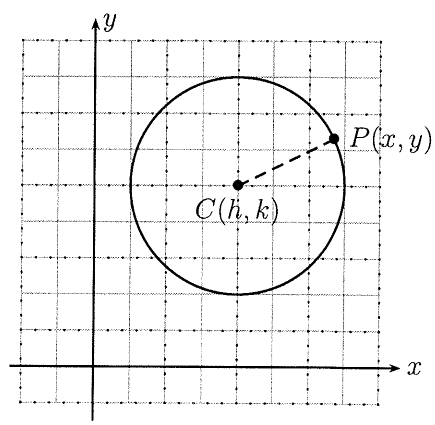
$y = -2$

$(0,-2)$

x	y
4	2
-4	2

By definition, a circle is the set of all points $P(x, y)$ whose distance from the center $C(h, k)$ is r. Thus, P is on the circle if and only if $\text{dist}(P, C) = r$. From the distance formula we have

$$\sqrt{(x-h)^2 + (y-k)^2} = r \quad \Leftrightarrow \quad (x-h)^2 + (y-k)^2 = r^2$$



▶ **Circles:** An equation of the circle with center $C(h, k)$ and radius r is

$$\boxed{(x-h)^2 + (y-k)^2 = r^2.}$$

This is called the **standard form** for the equation of the circle. If the center of the circle is the origin $(0, 0)$, then the equation is $x^2 + y^2 = r^2$.

Example 5: Find an equation of the circle such that:

$(x-h)^2+(y-k)^2=r^2$

$(x+1)^2+(y-4)^2=8^2$

$(x+1)^2+(y-4)^2=64$

- the radius is 8 and center is $(-1,4)$

- the endpoints of a diameter are $(-1,3)$ and $(7,-5)$.

CENTER

$M\left(\frac{7-1}{2}, \frac{-5+3}{2}\right)$

$M(3,-1)$ center

$d=\sqrt{(3+1)^2+(-1-3)^2}$

$=\sqrt{16+16}$

$=\sqrt{32}$

$(x-3)^2+(y+1)^2=(\sqrt{32})^2$

$(x-3)^2+(y+1)^2=32$

Example 6: Sketch the graph of the equation $x^2+y^2-4x+10y+13=0$ by showing that it represents a circle; then find its center and radius. Complete the square:

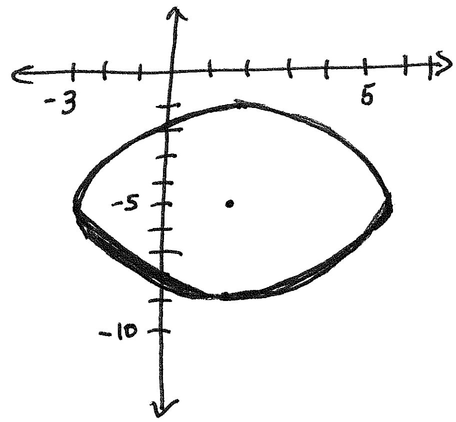
$x^2-4x+\_\_\_\_+y^2+10y+\_\_\_\_=-13+\_\_\_\_+\_\_\_\_$

$x^2-4x+\left(\frac{-4}{2}\right)^2+y^2+10y+\left(\frac{10}{2}\right)^2=-13+4+25$

$(x-2)^2+(y+5)^2=16$

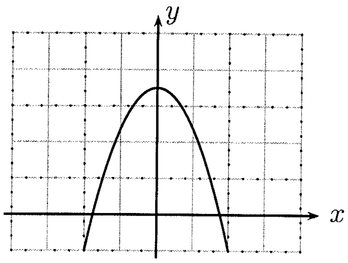
center $(h,k)=(2,-5)$

$r^2=16$, $r=4$

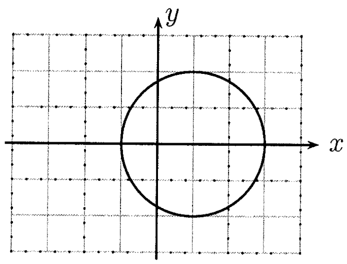


▶ **Symmetry:**

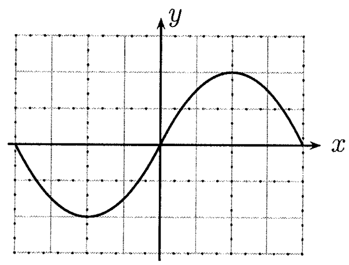
A graph is **symmetric with respect to the y-axis** if whenever $P(x,y)$ is on the graph, then so is $Q(-x,y)$.
A graph is **symmetric with respect to the x-axis** if whenever $P(x,y)$ is on the graph, then so is $Q(x,-y)$.
A graph is **symmetric with respect to the origin** if whenever $P(x,y)$ is on the graph, then so is $Q(-x,-y)$.



Symmetry w.r.t. the y-axis
Eq. is unchanged when $x \longleftrightarrow -x$
Graph is unchanged when reflected w.r.t. the y-axis



Symmetry w.r.t. the x-axis
Eq. is unchanged when $y \longleftrightarrow -y$
Graph is unchanged when reflected w.r.t. the x-axis



Symmetry w.r.t. the origin
Equation is unchanged when $x \longleftrightarrow -x$ AND $y \longleftrightarrow -y$
Graph is unchanged when rotated 180° about the origin

Example 7:

(a) The point $(2,3)$ is on a graph that is symmetric with respect to the y-axis.
The graph must also contain the point $(-2,3)$.

(b) The point $(2,3)$ is on a graph that is symmetric with respect to the x-axis.
The graph must also contain the point $(2,-3)$.

(c) The point $(2,3)$ is on a graph that is symmetric with respect to the origin.
The graph must also contain the point $(-2,-3)$.