MA109, Activity 9: Lines(Section 2.4, pp. 179-189) **Date:** _____

Today's Goal: We find equations for straight lines lying in a coordinate plane. The equation will depend on how the line is inclined, so we begin by discussing the concept of slope.

Assignments: **Homework (Sec. 2.4):** # 3, 6, 9, 11, 13, 16, 21, 27, 30, 41, 45, 48, 54, 57, 65 (pp. 189-192).

▶ The Slope of a Line:

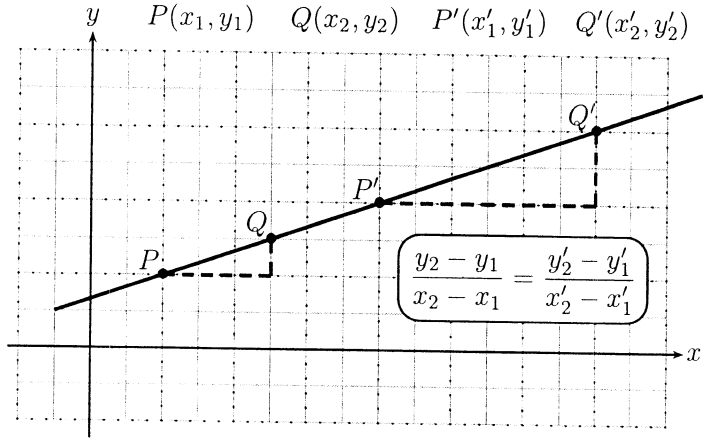
If a line lies in a coordinate plane, then the **run** is the change in the x-coordinate and the **rise** is the corresponding change in the y-coordinate between any two points on the line.

By definition, the slope m of a non-vertical line that passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

Note: Properties of similar triangles show that the slope is independent of which two points are chosen on the line.



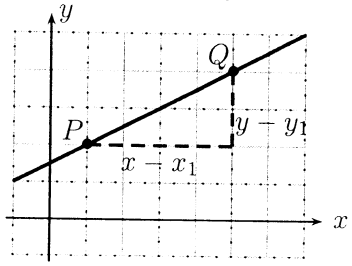
Example 1: Find the slope of the line that passes through the points $P(-1, -4)$ and $Q(6, 0)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{6 - (-1)} = \frac{0+4}{6+1} = \boxed{\frac{4}{7}}$ or $m = \frac{-4 - 0}{-1 - 6} = \frac{-4}{-7} = \frac{4}{7}$

▶ Equations of Lines:

Our goal is to find the equation of a line that passes through a given point $P(x_1, y_1)$ and has slope m. Any point $Q(x, y)$ with $x \neq x_1$ lies on this line if and only if the slope of the line through P and Q is equal to m:

$$\frac{y - y_1}{x - x_1} = m$$



Thus we have:

Point-Slope Form of the Equation of a Line:

The line that passes through the point $P(x_1, y_1)$ and has slope m has equation

$$\boxed{y - y_1 = m(x - x_1)}$$

Example 2:

Find an equation of the line through $P(-3, -5)$ and with slope $m = -7/2$.

Plug directly into point-slope equation

$y - (-5) = -7/2\,(x - (-3))$

$\boxed{y + 5 = -7/2\,(x + 3)}$

Example 3: Find an equation of the line through the points $P(3,2)$ and $Q(-3,4)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$.

Use either point in point-slope equation

$y - 2 = -\frac{1}{3}(x-3)$ or

$y - 4 = -\frac{1}{3}(x-(-3)) = -\frac{1}{3}(x+3)$

Slope-Intercept Form of the Equation of a Line:
The line that has slope m and y-intercept b has equation

$$y = mx + b$$

Vertical and Horizontal Lines: An equation of the vertical line through (a,b) is $x = a$

An equation of the horizontal line through (a,b) is $y = b$

General Equation of a Line: The graph of every **linear equation**

$$Ax + By + C = 0 \qquad (A, B \text{ not both zero})$$

is a line. Conversely, every line is the graph of a linear equation.

Example 4: Find an equation of the line with slope 2 and y-intercept -3.

Plug directly into slope-intercept equation

$y = mx + b \quad m = 2, \quad b = -3$

$y = 2x - 3$

Example 5: Find the slope and y-intercept of the line $4x + 5y = 10$.

Put equation into slope-intercept form by solving for y

$4x + 5y = 10$

$5y = 10 - 4x$

$y = \frac{10 - 4x}{5}$

$y = \frac{10}{5} - \frac{4x}{5}$

$y = -\frac{4x}{5} + 2$

slope $= -\frac{4}{5}$

y-intercept $= 2$

▶ Parallel and Perpendicular Lines:

Parallel Lines: Two non-vertical lines are parallel if and only if they have the same slope.

Perpendicular Lines: Two lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_1 m_2 = -1 \quad \Leftrightarrow \quad m_2 = -\frac{1}{m_1}.$$

Also, a horizontal line (slope 0) is perpendicular to a vertical line (no slope).

Example 6:

- Find an equation of the line that has y-intercept 6 and is parallel to the line $2x + 3y + 4 = 0$.

$2x + 3y + 4 = 0$

$3y = -2x - 4$

$y = -\frac{2}{3}x - 4/3$

slope $= -2/3$

using slope-intercept form

$y = mx + b$

since line is parallel to $2x + 3y + 4 = 0$

slope is the same, $m = -2/3$, $b = 6$

$y = -\frac{2}{3}x + 6$

- Find an equation of the line through $(-1, 2)$ and perpendicular to the line $4x - 8y = 1$.

$4x - 8y = 1$

$-8y = 1 - 4x$

$y = -\frac{1}{8} - \frac{4x}{-8}$

$y = \frac{1}{2}x - \frac{1}{8}$

slope $= \frac{1}{2}$

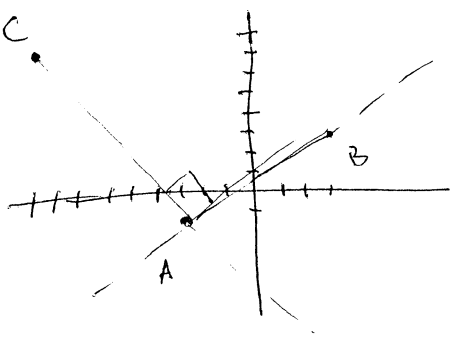
use point-slope form $y - y_1 = m(x - x_1)$. Since lines are perpendicular

$m_2 = \frac{-1}{m_1} = \frac{-1}{\frac{1}{2}} = -2$

$(y - 2) = -2(x - (-1))$

$y - 2 = -2(x + 1)$

Example 7: Show that $A(-3, -1)$, $B(3, 3)$ and $C(-9, 8)$ are vertices of a right triangle.



triangle is a right triangle if the line through points AC is perpendicular to line through AB.

slope of line through AC $m_2 = \frac{8 - (-1)}{-9 - (-3)} = \frac{9}{-6} = -\frac{3}{2}$

slope of line through AB $m_1 = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$

$\frac{2}{3} = \frac{-1}{-3/2}$ $\quad m_1 = -\frac{1}{m_2}$ so sides are perpendicular

▶ Applications: Slope as Rate of Change:

Example 8 (Global Warming):

Some scientists believe that the average surface temperature of the world has been rising steadily. The average surface temperature is given by $T = 0.02t + 8.50$, where T is the temperature in °C and t is years since 1900.

(*a*) What do the slope and T-intercept represent?

The rate of change in temperture is 0.02 degrees C per year

The temperture in 1900 ($t = 0$) was 8.50 °C

(*b*) Use the equation to predict the average global surface temperature in 2100.

2100 is 200 years after 1900 so $t = 200$

$T = 0.02(200) + 8.50 = 4 + 8.50 =$ 12.50 °C