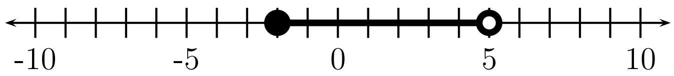
3 Solving Equations Practice Problems

1. Which of the following numbers is included in the graph? **The answers are in bold.**

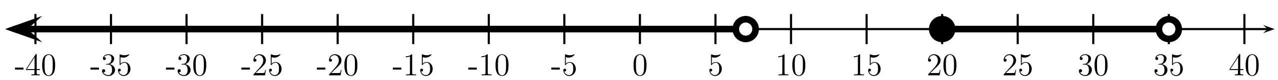


   (a) -5 (b) $\mathbf{-2}$ (c) $\mathbf{0}$ (d) 5 (e) 8

2. Which of the following numbers are included in the interval $(-\infty, 7) \cup [20, 35)$? **The answers are in bold.**

   (a) $\mathbf{-2,000,000}$ (b) $\mathbf{0}$ (c) **6.99999** (d) 7
   (e) 7.00000001 (f) 15 (g) 19.99999 (h) **20**
   (i) **20.00000001** (j) **24** (k) **34.99999** (l) 35
   (m) 35.00000001 (n) $2,000,000$

3. Sketch the graph of $(-\infty, 7) \cup [20, 35)$.



4. Find the exact value of $|\pi - 6|$. Your answer may not include absolute value symbols. **The answer is in bold.**

$$\mathbf{6} - \pi$$

5. Solve each equation or inequality algebraically. As you solve the equation or inequality, discuss the geometry (i.e., the number line) behind each step.

   (a) $|x - 7| = 5$

   Solution:$x = 2, 12$

   (b) $|2x + 5| - 3 = 1$

   Solution:$x = -\frac{9}{2}, -\frac{1}{2}$

   (c) $|x + 1| = |2x - 1|$

   Solution:$x = 0, 2$

   (d) $3|4x + 1| = 9$

   Solution:$x = -1, \frac{1}{2}$

   (e) $3|4 - x| + 6 = 2$

   No Real Solutions

6. Three pairs of equations are listed below. For each pair, determine if the two equations are equivalent. **The answers are in bold.**

   (a) $x + 5 = 2$ and $2x + 10 = 4$

   CIRCLE ONE: **EQUIVALENT** NOT EQUIVALENT

   (b) $x = 2$ and $x^2 = 4$

   CIRCLE ONE: EQUIVALENT **NOT EQUIVALENT**

   (c) $\frac{1}{x} = 5$ and $1 = 5x$

   CIRCLE ONE: **EQUIVALENT** NOT EQUIVALENT

7. Multiplying both sides of an equation by $x^2 + 1$ (**always**/sometimes/never) produces an equivalent equation. **The answer is in bold.**

8. Multiplying both sides of an equation by $|x|$ (always/**sometimes**/never) produces an equivalent equation. **The answer is in bold.**

9. Solve. (Describe the steps that are being applied to the variable. Think about how you will undo these to solve the equation.)

   (a) $4(x - 2)^2 - 3 = 0$

   $$\textbf{Solution:} x = 2 \pm \frac{\sqrt{3}}{2}$$

   (b) $4(x - 2)^2 + 3 = 0$

   No Real Solutions

   (c) $4(x - 2)^2 - 3 = 4x^2$

   $$\textbf{Solution:} x = \frac{13}{16}$$

   (d) $\frac{8-2s}{5} = 13$

   $$\textbf{Solution:} s = -\frac{57}{2}$$

   (e) $-5[14 - (3x + 1)^3] = 11$

   $$\textbf{Solution:} \frac{\sqrt[3]{16.2} - 1}{3}$$

10. Solve for a.

$$a + b = c(d + f)$$

$$\textbf{Solution:} a = c(d + f) - b$$

11. Solve for c.

$$a + b = c(d + f)$$

$$\textbf{Solution:} c = \frac{a + b}{d + f}$$

12. Solve for d.

$$a + b = c(d + f)$$

$$\textbf{Solution:} d = \frac{a+b}{c} - f$$

13. Solve for h.

$$V = \frac{\pi d^2 h}{4}$$

$$\textbf{Solution:} h = \frac{4V}{\pi d^2}$$

14. Solve for d.

$$V = \frac{\pi d^2 h}{4}$$

$$\textbf{Solution:} d = \pm\sqrt{\frac{4V}{\pi h}}$$

This is the formula for the volume of a cylinder. Does this simplify your solution? **If we know that this is the formula for the volume of a cylinder, then all the variables measure lengths and thus, we do not need to consider the negative square root.**

15. Solve.

(a) $\dfrac{3y^2 - 2y + 14}{y^2 + y - 2} = \dfrac{5}{y-1}$

$$\textbf{Solution:} y = \frac{4}{3}$$

(b) $\dfrac{x}{x+2} = \dfrac{5}{x} + 1$

$$\textbf{Solution:} x = -\frac{10}{7}$$

16. Use the Zero Product Property to solve the quadratic equation.

(a) $x^2 - 14 = 3x + 14$

$$\textbf{Solutions:} x = 4, 7$$

(b) $3x^2 + 16x + 5 = 0$

$$\textbf{Solutions:} x = -5, -\frac{1}{3}$$

17. Solve the quadratic equation by completing the square.

(a) $x^2 - 2x = 12$

$$\textbf{Solution:} x = \frac{2 \pm \sqrt{52}}{2} = 1 \pm \sqrt{13}$$

(b) $3x^2 = 12x + 1$

$$\textbf{Solution:} x = \frac{4 \pm \sqrt{52}}{6} = 2 \pm \frac{\sqrt{13}}{3}$$

18. How many solutions does each equation have?

$$\textbf{(I)}\ x^3 + 5 = 0 \qquad\qquad \textbf{(II)}\ x^4 = -4$$

Possibilities:

(a) Equation **(I)** has 3 solutions, and equation **(II)** has no solutions.

(b) Equation **(I)** has 3 solutions, and equation **(II)** has 1 solution.

(c) Equation **(I)** has 1 solution, and equation **(II)** has 2 solutions.

(d) Equation **(I)** has no solutions, and equation **(II)** has 2 solutions.

(e) Equation (I) has 1 solution, and equation (II) has no solutions.

19. Solve the quadratic equation by a method of your choice.

(a) $20x + 35 = 3x^2 + 4x$

$$\textbf{Solution:} x = \frac{16 \pm 26}{6} = 7, -\frac{5}{3}$$

(b) $7x^2 + x + 1 = 0$

No Real Solutions

20. Find a number k such that the equation has exactly one real solution.

$$x^2 + kx + 25 = 0$$

$$\textbf{Solution:} k = 10$$

21. Solve.

(a) $2x^6 = 9x^3 + 5$

$$\textbf{Solutions:} x = -\sqrt[3]{\frac{1}{2}}, \sqrt[3]{5}$$

(b) $3x^{1/2} + x^{1/4} - 10 = 0$

$$\textbf{Solutions:} x = \frac{625}{81}$$

(c) $t^3 - 2t^5 = 0$

$$\textbf{Solution:} t = 0, \pm\frac{1}{2}$$

(d) $\sqrt{3z - 5} = 3 - z$

$$\textbf{Solution:} z = 2$$

(e) $3\sqrt{t} + 10 = t$

$$\textbf{Solution:} t = 25$$

22. For each of the following equations, determine which technique you could use to solve the equation. There may be more than one or zero techniques.

   (a) $3 - x + 2x^2 = 5 + x$
   (b) $3x^5 - 7 = 2$
   (c) $x^5 + 3\sqrt{x} = 7$
   (d) $\dfrac{5}{x+2} - \dfrac{5+x}{2x} = \dfrac{7x}{x+2}$
   (e) $-4x + 3[5(x+7) - 3x + 2] = 7(x+5)$
   (f) $\dfrac{1}{x+2} = 5x$
   (g) $x^4 + 2x^2 - 1 = 0$
   (h) $x^4 + 2x - 1 = 0$
   (i) $x^4 + 2x = 0$

   On homework, quizzes, and exams, you will not be told which technique you should use. You should practice identifying techniques that can help you solve a problem.