MA 109 — College Algebra Spring 2011 **Name:** ______________ **Sec.:** ________
EXAM 1 - REVIEW

1. Which expression is equal to $x^2 + 12x + 31$? **HINT:** Complete the square.

Possibilities: leading coefficient is one ← GOOD

(a) $\boxed{(x+6)^2 - 5}$

(b) $(x+6)^2$

(c) $(x+12)^2$

(d) $(x+12)^2 + 31$

(e) $(x+6)^2 + 5$

$x^2 + 12x \quad +31$ ($b=12$)

Need $\left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = 6^2 = 36$

$= (x^2 + 12x + 36) - 36 + 31$

$= \left(x + \frac{12}{2}\right)^2 - 5$ ← Perfect Square factors to $\left(x + \frac{b}{2}\right)^2$

$= \boxed{(x+6)^2 - 5}$

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2. The area of the shaded region is 216 square feet. Find a.

Possibilities:

(a) $6\sqrt{6}$ feet

(b) $\boxed{12 \text{ feet}}$

(c) 24 feet

(d) $6\sqrt{2}$ feet

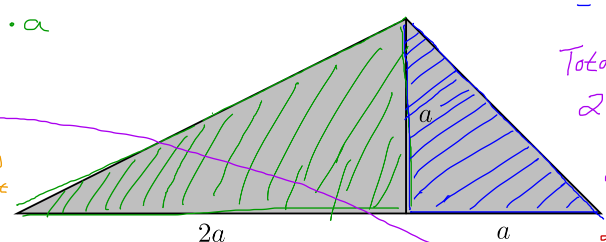
(e) 18 feet

Recall: Area of a triangle $= \frac{1}{2}$ base · height

Area $= \frac{1}{2} 2a \cdot a$

$= a \cdot a$

$= a$



Area $= \frac{1}{2} \cdot a \cdot a$

$= \frac{1}{2} a^2$

Total Area $= a^2 + \frac{1}{2}a^2$

$216 = \frac{2a^2}{2} + \frac{a^2}{2}$

$216 = \frac{3a^2}{2}$ Multiply by 2

$2(216) = 2\left(\frac{3a^2}{2}\right)$ Simplify

$432 = 3a^2$ Divide by 3

$\frac{432}{3} = \frac{3a^2}{3}$ Simplify

$144 = a^2$ Square Root

$\sqrt{144} = \sqrt{a^2}$ Simplify

$12 = |a|$ $a \geq 0$ since a represents a distance

$\boxed{a = 12 \text{ feet}}$

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3. Which of the following ALWAYS produces an equivalent equation?

Possibilities:

(a) Multiplying both sides of an equation by 9.

~~(b)~~ Dividing both sides of the equation by x.

~~(c)~~ Squaring both sides of an equation.

(d) Multiplying both sides of an equation by $x + 9$.

~~(e)~~ Taking the absolute value of both sides of an equation.

Suppose $x = 1$ one solution of 1

then $x \cdot x = x \cdot 1$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ or $x - 1 = 0$

two solutions $x = 1$

$x = 0$ one solution of zero

$x(x+9) = 0 \cdot (x+9)$

$x(x+9) = 0$ so $x = 0$ or $x + 9 = 0$

two solutions $x = -9$

$x = -1$ ← one solution of -1

$x^2 = (-1)^2$

$x^2 = 1$ so $x = \pm 1$ two solutions

$x^2 = -1$ ← no solution

$|x^2| = |-1|$

$x^2 = 1$ so $x = \pm 1$ ← two solutions

4. Find all real solutions or state that there are NONE.

$$\sqrt{x+2} = x+5.$$

No Real Solutions
work on Next page

5. Find all real solutions or state that there are NONE.

$$(x+2)^4 + 10 = -6$$

No Real Solutions
Work on Next Page

6. Solve the equation for a.

$$-7x - 49 = 2 + a.$$ Subtract 2

$-7x-49-2 = 2+a-2$ Simplify

$-7x-51 = a$

so $\boxed{a = -7x-51}$

7. Find the slope of the line through the points $(\underset{x_1}{-1}, \underset{y_1}{4})$ and $(\underset{x_2}{11}, \underset{y_2}{-6})$.

slope $= m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-6-4}{11-(-1)} = \dfrac{-10}{11+1} = -\dfrac{10}{12} = \boxed{\dfrac{-5}{6}}$

8. The endpoints of a diameter of a circle are $A(\underset{x_1}{8}, \underset{y_1}{-9})$ and $B(\underset{x_2}{-5}, \underset{y_2}{7})$. Find the center of the circle.

Possibilities:

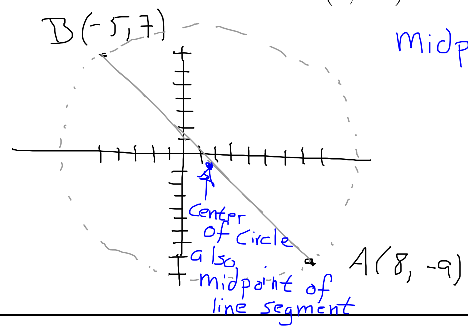
(a) $(0, 5\sqrt{17})$

(b) $(13/2, -8)$

(c) $(-13/2, 8)$

(d) $(5\sqrt{17}, 0)$

(e) $\boxed{(3/2, -1)}$



Midpoint $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{8+(-5)}{2}, \dfrac{-9+7}{2}\right)$

$= \left(\dfrac{3}{2}, \dfrac{-2}{2}\right) =$

$= \left(\dfrac{3}{2}, -1\right)$

9. Find all real solutions or state that there are NONE.

$$x^3 + 5 = 13$$ Subtract 5

$x^3+5-5 = 13-5$ Simplify

$x^3 = 8$ Cube root

$\sqrt[3]{x^3} = \sqrt[3]{8}$ Simplify

$x = 2$ Check

$2^3+5 = 8+5 = 13$ ☺ Good!

$\boxed{x = 2 \text{ is the only real solution}}$

10. Find all real solutions or state that there are NONE.

$$\frac{x-6}{4} = -9.$$ Multiply by 4

$4\left(\dfrac{x-6}{\cancel{4}}\right) = 4(-9)$ Simplify

$x-6 = -36$ Add 36

$x-6+6 = -36+6$ Simplify

$x = -30$ Check

$\dfrac{-30-6}{4} = \dfrac{-36}{4}$

11. Find all real solutions.

$$x^3 + 12x^2 + 25x + 200 = 4x^2$$ Subtract $4x^2$

$x^3+12x^2+25x+200-4x^2 = 4x^2-4x^2$ Simplify

Possibilities:

(a) $\boxed{x = -8}$

(b) $x = 5$ and $x = 8$

(c) $x = 5$ and $x = -5$

(d) $x = 8$

(e) $x = 5$, $x = -5$, and -$x = 8$

$x^3+8x^2+25x+200 = 0$ Factor by grouping

$x^2(x+8)+25(x+8) = 0$

$(x^2+25)(x+8) = 0$ Zero Product Rule

$x^2+25 = 0$ or $x+8 = 0$

$x^2 = -25$ $x = -8$

Impossible   Check

$(-8)^3+12(-8)^2+25(-8)+200 \stackrel{?}{=} 4(-8)^2$

$-512+12(64)-200+200 \stackrel{?}{=} 4(64)$

$-512+768 \stackrel{?}{=} 256$

$256 = 256$

☺ Consequently $x=-8$ is the only real solution

#4

$\sqrt[2]{x+2} = x+5$ — $\sqrt[n]{a} = b$ means $b^n = a$

$(x+5)^2 = x+2$ — Simplify

$(x+5)(x+5) = x+2$

$x^2 + 5x + 5x + 25 = x+2$

$x^2 + 10x + 25 = x+2$ — Subtract x

$x^2 + 10x + 25 - x = x + 2 - x$ — Simplify

$x^2 + 9x + 25 = 2$ — Subtract 2

$x^2 + 9x + 25 - 2 = 2 - 2$ — Simplify

$x^2 + 9x + 23 = 0$ — Factor

Need two numbers that:

① multiply to $a \cdot c = 1 \cdot 23 = 23$ ← Prime

② Add to $b = 9$

So we must use the Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm \sqrt{9^2 - 4(1)(23)}}{2(1)}$$

$$= \frac{-9 \pm \sqrt{81 - 92}}{2} = \frac{-9 \pm \sqrt{-11}}{2}$$

NOTE: the discriminant $b^2 - 4ac$ is negative so there are

$\boxed{\text{no real solutions}}$

#5

$(x+2)^4 + 10 = -6$ — Subtract 10

$(x+2)^4 + 10 - 10 = -6 - 10$ — Simplify

$(x+2)^4 = -16$

Impossible

any real number raised to an even exponent will not result in a negative value

12. How many solutions are there for each equation?

Note: odd exponent　　　　Note: even exponent

(I) $(x-2)^3 = 3$　　　　**(II)** $(x+5)^2 = 1$

Possibilities:　one solution　$x-2 = \sqrt[3]{3}$　$x = 2 + \sqrt[3]{3}$　　$x+5 = \pm\sqrt[2]{1}$　$x+5 = \pm 1$

(a) Equation **(I)** has 3 solutions, and equation **(II)** has 2 solutions.　$x = -5 \pm 1$

(b) Equation **(I)** has no solutions, and equation **(II)** has no solutions.　$x = -4$ and $x = -6$

(c) Equation **(I)** has 3 solutions, and equation **(II)** has 1 solution.　Two Solutions

(d) Equation **(I)** has 1 solution, and equation **(II)** has 2 solutions.

(e) Equation **(I)** has 1 solution, and equation **(II)** has no solutions.

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13. Find the perimeter of the parallelogram ABCD with vertices A(1, 4), B(18, 4), C(21, 8), and D(4, 8).

Possibilities:

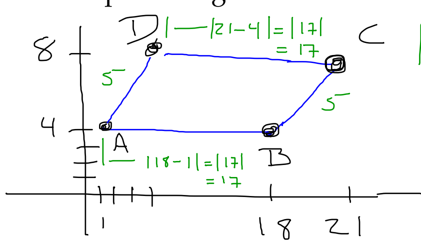
(a) 44 units

(b) 34 units

(c) 68 units

(d) 42 units

(e) 378 units



$|AD| = \sqrt{(1-4)^2 + (4-8)^2} = \sqrt{(-3)^2 + (-4)^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

$|BC| = \sqrt{(18-21)^2 + (4-8)^2} = \sqrt{(-3)^2 + (-4)^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

Perimeter $= 5 + 17 + 5 + 17 = 44$ units

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14. A circle has a diameter with endpoints (16,2) and (4,-8). Find an equation for the circle.

($x_1\ y_1$, $x_2\ y_2$)

Standard Form for Equation of a circle $(x-h)^2 + (y-k)^2 = r^2$

Possibilities:

Center of circle = midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{16+4}{2}, \frac{2+(-8)}{2}\right) = \left(\frac{20}{2}, \frac{-6}{2}\right) = (10, -3)$ (h, k)

$(x-10)^2 + (y-(-3))^2 = (\sqrt{61})^2$

$\boxed{(x-10)^2 + (y+3)^2 = 61}$

(a) $(x-10)^2 + (y+3)^2 = \sqrt{61}$

(b) $(x-6)^2 + (y-5)^2 = 61$

(c) $(x+10)^2 - (y-3)^2 = 61$

(d) $(x-6)^2 + (y-5)^2 = \sqrt{61}$

(e) $(x-10)^2 + (y+3)^2 = 61$

radius = distance from center (10, -3) to any endpoint (16, 2)

$= \sqrt{(10-16)^2 + (-3-2)^2} = \sqrt{(-6)^2 + (-5)^2} = \sqrt{36+25}$
$= \sqrt{61}$

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15. Find all real solutions or state that there are NONE.

$$-4x + 16 = -5x + 3.$$　Subtract 16

$-4x + 16 - 16 = -5x + 3 - 16$　Simplify

$-4x = -5x - 13$　Add $5x$

$-4x + 5x = -5x - 13 + 5x$　Simplify

$x = -13$　Check

$-4(-13) + 16 \stackrel{?}{=} -5(-13) + 3$

$52 + 16 \stackrel{?}{=} 65 + 3$

$68 = 68$ ☺

Consequently $x = -13$ is the only solution

16. Find the y-intercept of the graph of $y = 2x^7 - 493x^5 + 969$.

y-intercept occur when $x=0$
so $y = 2(0)^7 - 493(0)^5 + 969 = 969$
so the y-intercept is $(0,969)$ or 969

17. Solve the equation for a.

$$P = 4a + 5b.$$ Subtract 5b

$P - 5b = 4a + 5b - 5b$ Simplify

$P - 5b = 4a$ Divide by 4

$\frac{P-5b}{4} = \frac{4a}{4}$ Simplify

$\boxed{a = \frac{P-5b}{4}}$

18. Find all the solutions of the equation.

$$|x+4| = 8$$

Possibilities:

(a) The only solution is $x = 4$.
(b) There are exactly two solutions: $x = 4$ and $x = -4$.
(c) The only solution is $x = -12$.
(d) There are exactly two solutions: $x = 4$ and $x = -12$.
(e) The equation does not have any solutions.

$x + 4 = 8$
$-4 \quad -4$
$x = 4$

and

$x + 4 = -8$
$-4 \quad -4$
$x = -12$

Check $x = 4$
$|4+4| \stackrel{?}{=} 8$
$|8| \stackrel{?}{=} 8$
$8 = 8$ ☺

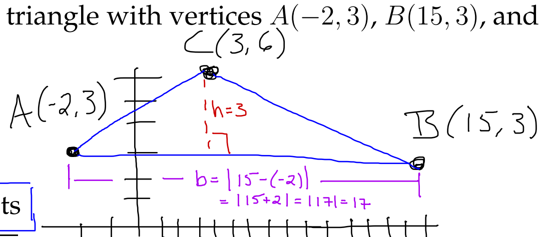
Check $x = -12$
$|-12+4| \stackrel{?}{=} 8$
$|-8| \stackrel{?}{=} 8$
$8 = 8$ ☺

Consequently $x=4$ and $x=-12$ are both solutions

19. Find the area of the triangle with vertices $A(-2, 3)$, $B(15, 3)$, and $C(3, 6)$.

Possibilities:

(a) 51 square units
(b) 54 square units
(c) $\boxed{51/2 \text{ square units}}$
(d) $17 + 3\sqrt{17} + \sqrt{34}$ square units
(e) 90 square units



Area of a triangle
$= \frac{1}{2} b h$
$= \frac{1}{2} \cdot 17 \cdot 3$
$= \boxed{\frac{51}{2} \text{ square units}}$

20. Find all real solutions or state that there are NONE.

$$x^2 + 18x + 63 = 2x.$$

$x = -7$ and $x = -9$ See work on Next Page

21. $\boxed{\textbf{TRUE}}$ **or FALSE:** The line through the points $(1, 1)$ and $(4, -5)$ is perpendicular to the line through the points $(-9, -1)$ and $(-15, -4)$.

$x_1\ y_1 \qquad x_2\ y_2$

$x_1, y_1 \qquad x_2, y_2$

$\text{Slope}_1 = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-1}{4-1} = \frac{-6}{3} = -2$ Note $\perp m_1 = +\frac{1}{2} = m_2$

$\text{Slope}_2 = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-1)}{-15-(-9)} = \frac{-4+1}{-15+9} = \frac{-3}{-6} = \frac{1}{2}$

4

$\boxed{\#20}$ $x^2+18x+63=2x$ Subtract $2x$

$x^2+18x+63-2x=2x-2x$ Simplify

$x^2+16x+63=0$

Discriminant $= b^2-4ac = 16^2-4(1)(63)$

$= 256-252$

$= 4$

Quadratic Formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-16\pm\sqrt{4}}{2(1)} = \dfrac{-16\pm 2}{2} =$

$x=\dfrac{-16+2}{2}=\dfrac{-14}{2}=\boxed{-7}$ and $x=\dfrac{-16-2}{2}=\dfrac{-18}{2}=\boxed{-9}$

Check $x=-7$

$(-7)^2+18(-7)+63 \overset{?}{=} 2(-7)$

$49-126+63 \overset{?}{=} -14$

$-77+63 \overset{?}{=} -14$

$-14=-14$ ☺

Check $x=-9$

$(-9)^2+18(-9)+63 \overset{?}{=} 2(-9)$

$81-162+63 \overset{?}{=} -18$

$-81+63 \overset{?}{=} -18$

$-18=-18$ ☺

Consequently $x=-7$ is a solution

Consequently $x=-9$ is a solution

22. Find an equation for the line through the points $(\overset{x_1}{-2}, \overset{y_1}{2})$ and $(\overset{x_2}{7}, \overset{y_2}{11})$.

Possibilities:

Point-Slope Form of a line $y - y_1 = m(x - x_1)$

(a) $y - 2 = -\frac{9}{9}(x-2)$

$\boxed{\text{(b) } y - 2 = \frac{9}{9}(x+2)}$

~~(c)~~ $y + 2 = \frac{9}{9}(x-2)$

(d) $y - 11 = -\frac{9}{9}(x-7)$

~~(e)~~ $y - 7 = \frac{9}{9}(x-11)$

1st Compute the slope

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-2}{7-(-2)} = \frac{11-2}{7+2} = \frac{9}{9} = 1$

Using point $(-2, 2)$ yields $y - 2 = \frac{9}{9}(x - (-2))$

$y - 2 = \frac{9}{9}(x+2)$

Using point $(7, 11)$ yields $y - 11 = \frac{9}{9}(x-7)$

Suggestion: Write this step to see the correct answer ☺

23. Find all real solutions or state that there are NONE.

$$x^4 - 10x^2 + 9 = 0.$$

The real solutions are $x=1$, $x=-1$, $x=3$ and $x=-3$

See next page for work

24. Simplify.

$$(5x+6)(2x-5) - 16x - 56$$ Distribute (FOIL)

$10x^2 - 25x + 12x - 30 - 16x - 56$ Combine like terms

$10x^2 - 29x - 86$

Possibilities:

(a) $10x^2 - 13x - 22$

(b) $17x + 3$

$\boxed{\text{(c) } 10x^2 - 29x - 86}$

(d) $10x - 38$

(e) $10x^2 - 29x + 26$

25. Find all real solutions or state that there are NONE.

$$x^2 + 3x - 20 = 8.$$

The real solutions are $x = -7$ and $x = 4$

See next page for work.

26. Find all real solutions or state that there are NONE.

$$\frac{2}{x+8} + \frac{5}{x-9} = \frac{3}{x^2 - x - 72}.$$

$x = \frac{-19}{7}$ is the only real solution See following pages for work

27. Find all real solutions or state that there are NONE.

$$x^3 + x + 1 = x + 28.$$ Subtract x

$x^3 + x + 1 - x = x + 28 - x$ Simplify

$x^3 + 1 = 28$ Subtract 1

$x^3 + 1 - 1 = 28 - 1$ Simplify

$x^3 = 27$

$x = \sqrt[3]{27} = 3$ Check

Check $x = 3$

$3^3 + 3 + 1 \overset{?}{=} 3 + 28$

$27 + 3 + 1 \overset{?}{=} 31$

$30 + 1 \overset{?}{=} 31$

$31 = 31$

Consequently, $x = 3$ is the only real solution

5

#23

$x^4 - 10x^2 + 9 = 0$ Let $u = x^2$

$u^2 - 10u + 9 = 0$ then $u^2 = (x^2)^2$ ← recall $(a^m)^n = a^{m \cdot n}$

$u^2 - 9u - u + 9 = 0$ so $u^2 = x^4$

$u(\underline{u-9}) - 1(\underline{u-9}) = 0$

$(u-1)(u-9) = 0$ Zero Product Rule

$u - 1 = 0$ or $u - 9 = 0$

$+1 \quad +1$ $\quad +9 \quad +9$

Replace u with x^2

$u = 1$ $\quad u = 9$

$x^2 = 1$ $\quad x^2 = 9$

$x = \pm\sqrt{1}$ $\quad x = \pm\sqrt{9}$

$x = \pm 1$ $\quad x = \pm 3$

Check $x = 1$

$1^4 - 10(1)^2 + 9 \stackrel{?}{=} 0$

$1 - 10 + 9 \stackrel{?}{=} 0$

$-9 + 9 \stackrel{?}{=} 0$

$0 = 0$ ☺

So $x = 1$ is a solution

Check $x = -1$

$(-1)^4 - 10(-1)^2 + 9 \stackrel{?}{=} 0$

$1 - 10 + 9 \stackrel{?}{=} 0$

$-9 + 9 \stackrel{?}{=} 0$

$0 = 0$ ☺

So $x = -1$ is a solution

Check $x = 3$

$(3)^4 - 10(3)^2 + 9 \stackrel{?}{=} 0$

$81 - 10 \cdot 9 + 9 \stackrel{?}{=} 0$

$81 - 90 + 9 \stackrel{?}{=} 0$

$-9 + 9 \stackrel{?}{=} 0$

$0 = 0$ ☺

So $x = 3$ is a solution

Check $x = -3$

$(-3)^4 - 10(-3)^2 + 9 = 0$

$81 - 10 \cdot 9 + 9 \stackrel{?}{=} 0$

$81 - 90 + 9 \stackrel{?}{=} 0$

$-9 + 9 \stackrel{?}{=} 0$

$0 = 0$ ☺

So $x = -3$ is a solution

#25

$x^2 + 3x - 20 = 8$ Subtract 8

$x^2 + 3x - 20 - 8 = 8 - 8$ Simplify

$x^2 + 3x - 28 = 0$ Factor

$x^2 - 4x + 7x - 28 = 0$

$x(\underline{x-4}) + 7(\underline{x-4}) = 0$

$(x+7)(x-4) = 0$ Zero Product Rule

$x + 7 = 0$ or $x - 4 = 0$

$-7 \quad -7$ $\quad +4 \quad +4$

$x = -7$ $\quad x = 4$

Check $x = -7$

$(-7)^2 + 3(-7) - 20 \stackrel{?}{=} 8$

$49 - 21 - 20 \stackrel{?}{=} 8$

$28 - 20 \stackrel{?}{=} 8$

$8 = 8$ ☺

So $x = -7$ is a solution

Check $x = 4$

$4^2 + 3(4) - 20 \stackrel{?}{=} 8$

$16 + 12 - 20 \stackrel{?}{=} 8$

$28 - 20 \stackrel{?}{=} 8$

$8 = 8$

So $x = 4$ is a solution.

$\boxed{\#26}$ $$\frac{2}{x+8}+\frac{5}{x-9}=\frac{3}{x^2-x-72}$$

Compute the LCD:

① The Denominators are

$x+8$

$x-9$

$x^2-x-72=(x+8)(x-9)$

$x^2-x-72=x^2-9x+8x-72$

$=x(x-9)+8(x-9)$

$=(x+8)(x-9)$

② LCD $=(x+8)^1(x-9)^1$

Multiply both sides by the LCD $(x+8)(x-9)$

$$\boxed{(x+8)(x-9)}\left(\frac{2}{x+8}+\frac{5}{x-9}\right)=\left(\frac{3}{\cancel{(x+8)}\cancel{(x-9)}}\right)\cancel{(x+8)}\cancel{(x-9)}$$

$$\cancel{(x+8)}(x-9)\frac{2}{\cancel{x+8}}+(x+8)\cancel{(x-9)}\frac{5}{\cancel{x-9}}=3$$ Simplify

$$(x-9)2+(x+8)5=3$$ Distribute

$$2x-18+5x+40=3$$ Simplify

$$7x+22=3$$ Subtract 22

$$7x+22-22=3-22$$ Simplify

$$7x=-19$$ Divide by 7

$$x=\frac{-19}{7}$$

Check $x=\frac{-19}{7}$

$$\frac{2}{-\frac{19}{7}+\frac{8}{1}\cdot\frac{7}{7}}+\frac{5}{\frac{-19}{7}-\frac{9}{1}\cdot\frac{7}{7}}\overset{?}{=}\frac{3}{\left(\frac{-19}{7}\right)^2-\left(\frac{-19}{7}\right)-72}$$

$$\frac{2}{\frac{-19+56}{7}}+\frac{5}{\frac{-19-63}{7}}\overset{?}{=}\frac{3}{\frac{361}{49}+\frac{19}{7}\cdot\frac{7}{7}-\frac{72}{1}\cdot\frac{49}{49}}$$

$$\frac{2/1}{\frac{37}{7}}+\frac{5/1}{\frac{-82}{7}}\overset{?}{=}\frac{3}{\frac{361+133-3528}{49}}$$

$$\frac{2}{1}\cdot\frac{7}{37}-\frac{5}{1}\cdot\frac{7}{82}\overset{?}{=}\frac{3/1}{\frac{-3034}{49}}$$

$$\frac{82}{82}\,\frac{14}{37}-\frac{37}{37}\,\frac{35}{82}\overset{?}{=}-\frac{3}{1}\cdot\frac{49}{3034}$$

$$\frac{1148}{3034}-\frac{1295}{3034}\overset{?}{=}\frac{-147}{3034}$$

$$\frac{-147}{3034}=\frac{-147}{3034}$$ ☺ So $x=\frac{-19}{7}$ is the only real solution

28. Solve.

$$\frac{2}{x} + \frac{7}{x-8} = 0$$

1st Compute the LCD
The denominators are
x
$x-8$
$LCD = x(x-8)$

Multiply by LCD

$$\boxed{x(x-8)}\left[\frac{2}{x} + \frac{7}{x-8}\right] = x(x-8)\,0$$

Simplify

$$\cancel{x}(x-8)\frac{2}{\cancel{x}} + x\cancel{(x-8)}\frac{7}{\cancel{x-8}} = 0$$

$$(x-8)\cdot 2 + x\cdot 7 = 0$$

$$2x - 16 + 7x = 0$$

$$9x - 16 = 0 \quad \text{Add } 16$$

$$9x - 16 + 16 = 0 + 16 \quad \text{Simplify}$$

$$9x = 16 \quad \text{Divide by } 9$$

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{16}{9} \quad \text{Simplify}$$

$$\boxed{x = \frac{16}{9}} \quad \text{Check}$$

Possibilities:

(a) -8/5
(b) -16/5
(c) 8/9
(d) 56/9
$\boxed{\text{(e) } 16/9}$

29. Which of the following equations are linear equations?

Equivalent to $Ax + By = C$

(I) $y = \dfrac{7}{x+2}$ $\times$ **(II)** $y - 3 = 9(x-2)$ ✓ **(III)** $y = \pi x - 2$ ✓

$y(x+2) = 7$

$xy + 2y = 7$

Possibilities:

(a) Only equation **(II)** is linear.
(b) None of the equations are linear.
(c) Only equations **(I)** and **(II)** are linear
(d) All of the equations are linear.
$\boxed{\text{(e)}}$ Only equations **(III)** and **(III)** are linear

30. The distance from x to 6 is 4. Which of the following equations represents this fact?

Possibilities:

$\boxed{\text{(a) } |x-6| = 4}$
(b) $|x+6| = 4$
(c) $|x+4| = 6$
(d) $|x-4| = 6$
(e) $|x| = 10$

The distance from a to b is $|a-b|$
So the distance from x to 6 is $|x-6|$
therefore the above says $|x-6| = 4$
the distance from x to 6 is 4

Recall: $|x-y| = |y-x|$ Consequently, $|6-x| = 4$ would also be correct.

31. Find all real solutions or state that there are NONE.

$$(x-1)^2 - 6x = (x-7)^2 + 3$$

$x = \frac{17}{2}$ is the only real solution, see work on Next page.

$\boxed{\#28}$ Check $x = \frac{16}{9}$

$$\frac{2/1}{\frac{16}{9}} + \frac{7}{\frac{16}{9} - \frac{8}{1}\cdot\frac{9}{9}} \stackrel{?}{=} 0$$

$$\frac{\cancel{2}^{1}}{1} \cdot \frac{9}{\cancel{16}_{8}} + \frac{7}{\frac{16-72}{9}} \stackrel{?}{=} 0$$

$$\frac{9}{8} + \frac{7/1}{\frac{-56}{9}} \stackrel{?}{=} 0$$

$$\frac{9}{8} - \frac{\cancel{7}^{1}}{1} \cdot \frac{9}{\cancel{56}_{8}} \stackrel{?}{=} 0$$

$$\frac{9}{8} - \frac{9}{8} \stackrel{?}{=} 0$$

$$0 = 0 \quad ☺$$

Consequently, $x = \frac{16}{9}$ is the <u>only</u> real solution

$\boxed{\#31}$

$$(x-1)^2 - 6x = (x-7)^2 + 3 \qquad a^2 = a \cdot a$$

$$(x-1)(x-1) - 6x = (x-7)(x-7) + 3 \qquad \text{Distribute (FOIL)}$$

$$x^2 - x - x + 1 - 6x = x^2 - 7x - 7x + 49 + 3 \qquad \text{Simplify}$$

$$x^2 - 8x + 1 = x^2 - 14x + 52 \qquad \text{Subtract } x^2$$

$$\cancel{x^2} - 8x + 1 - \cancel{x^2} = \cancel{x^2} - 14x + 52 - \cancel{x^2} \qquad \text{Simplify}$$

$$-8x + 1 = -14x + 52 \qquad \text{Add } 14x$$

$$-8x + 1 + 14x = -14x + 52 + 14x \qquad \text{Simplify}$$

$$6x + 1 = 52 \qquad \text{Subtract } 1$$

$$6x + 1 - 1 = 52 - 1 \qquad \text{Simplify}$$

$$6x = 51 \qquad \text{Divide by } 6$$

$$\frac{\cancel{6}x}{\cancel{6}} = \frac{51}{6} \qquad \text{Simplify}$$

$$x = \frac{51}{6} = \boxed{\frac{17}{2}} \qquad \text{Check}$$

Check $x = \frac{17}{2}$

$$\left(\frac{17}{2} - 1\right)^2 - \cancel{6}^{3} \cdot \frac{17}{\cancel{2}} \stackrel{?}{=} \left(\frac{17}{2} - \frac{7}{1}\right)^2 + 3$$

$$\left(\frac{17}{2} - \frac{2}{2}\right)^2 - 51 \stackrel{?}{=} \left(\frac{17}{2} - \frac{14}{2}\right)^2 + 3$$

$$\left(\frac{15}{2}\right)^2 - 51 \stackrel{?}{=} \left(\frac{3}{2}\right)^2 + 3$$

$$\frac{225}{4} - \frac{51}{1} \cdot \frac{4}{4} \stackrel{?}{=} \frac{9}{4} + \frac{3}{1} \cdot \frac{4}{4}$$

$$\frac{225 - 204}{4} \stackrel{?}{=} \frac{9 + 12}{4}$$

$$\frac{21}{4} = \frac{21}{4} \quad ☺$$

Consequently, $x = \frac{17}{2}$ is a solution

32. Find all real solutions.

$$x^3 + 6x^2 + x - 6 = 2x$$

$X^3+6x^2+x-6-2x=2x-2x$ Subtract 2x

$x^3+6x^2-x-6=0$ Simplify

$x^2(x+6)-1(x+6)=0$ Factor by Grouping

$(x^2-1)(x+6)=0$ Zero Product Rule

$x^2-1=0$ or $x+6=0$

$+1\quad +1 \qquad -6\quad -6$

$x^2=1 \qquad x=-6$

$x=\pm\sqrt{1}$ See Next Page for check

$x=\pm 1$

Possibilities:

(a) $x = 1$ and $x = 6$

(b) $x = 1$ and $x = -6$

(c) $x = -6$

(d) $x = 1$, $x = -1$, and $x = -6$

(e) $x = 1$, $x = -1$, and $x = 6$

33. Find an equation for the line that is perpendicular to $y = \frac{5}{6}x + 6$ and contains the point (0,12).

$y=-\frac{6}{5}x+12$ See work on Next Page

34. How many solutions does the equation have?

$$-3x^2 + 12x = -3$$

$X=2\pm\sqrt{5}$ See Next Page for work

35. Find all real solutions.

$$x = 2x^2$$

$X-x=2x^2-x$ Subtract x

$0=2x^2-x$ Simplify

$0=x(2x-1)$ Factor

Zero Product Rule

$X=0$ or $2x-1=0$

$2x=1$

$x=\frac{1}{2}$

Check $x=0$

$0 \stackrel{?}{=} 2(0)^2$

$0=0$ ☺

So $x=0$ is a solution

Check $x=\frac{1}{2}$

$\frac{1}{2} \stackrel{?}{=} 2\left(\frac{1}{2}\right)^2$

$\frac{1}{2} \stackrel{?}{=} 2\cdot\frac{1}{4}$

$\frac{1}{2}=\frac{1}{2}$ ☺

So $x=\frac{1}{2}$ is a solution

Possibilities:

$\boxed{\text{(a) } x = 0 \text{ and } x = \tfrac{1}{2}}$

(b) $x = 0$ and $x = 2$

(c) $x = 0$

(d) $x = \frac{1}{2}$

(e) $x = 2$

#32 Check $x=1$

$$1^3+6\cdot1^2+1-6 \stackrel{?}{=} 2(1)$$
$$1+6+1-6 \stackrel{?}{=} 2$$
$$7+1-6 \stackrel{?}{=} 2$$
$$8-6 \stackrel{?}{=} 2$$
$$2=2$$

So $x=1$ is a solution

Check $x=-1$

$$(-1)^3+6(-1)^2+(-1)-6 \stackrel{?}{=} 2(-1)$$
$$-1+6-1-6 \stackrel{?}{=} -2$$
$$5-1-6 \stackrel{?}{=} -2$$
$$4-6 \stackrel{?}{=} -2$$
$$-2=-2$$

So $x=-1$ is a solution

Check $x=-6$

$$(-6)^3+6(-6)^2+(-6)-6 \stackrel{?}{=} 2(-6)$$
$$-216+6\cdot36-6-6 \stackrel{?}{=} -12$$
$$-216+216-6-6 \stackrel{?}{=} -12$$
$$-6-6 \stackrel{?}{=} -12$$
$$-12=-12$$

So $x=-6$ is a solution

#33 $y=\frac{5}{6}x+6$ has slope $=\frac{5}{6}$

the perpendicular to this slope is $-\frac{6}{5}$

Consequently an equation of a line with slope $-\frac{6}{5}$ containing the point $(0,12)$ is $\boxed{y=-\frac{6}{5}x+12}$

y-intercept

$y=mx+b$ slope-intercept form

Note if you are not given the y-intercept then use point-slope form of a line: $y-y_1=m(x-x_1)$

#34

$$-3x^2+12x=-3 \qquad \text{Divide by } -3$$
$$\frac{-3x^2}{-3}+\frac{12x}{-3}=\frac{-3}{-3} \qquad \text{Simplify}$$
$$x^2-4x=1 \qquad \text{Subtract } 1$$
$$x^2-4x-1=1-1 \qquad \text{Simplify}$$
$$x^2-4x-1=0$$

Discriminant $= b^2-4ac=(-4)^2-4(1)(-1)$
$= 16+4 = 20$

20 → 2, 10; 10 → 5, 2

So $\sqrt{20}=\sqrt{2^2\cdot5}=\sqrt{2^2}\cdot\sqrt{5}=2\sqrt{5}$

Quadratic Formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-4)\pm\sqrt{20}}{2(1)}=\dfrac{4\pm\sqrt{20}}{2}$

$$=\frac{4\pm2\sqrt{5}}{2}=\frac{\cancel{2}(2\pm\sqrt{5})}{\cancel{2}}=\boxed{2\pm\sqrt{5}}$$

36. Find all the solutions of the equation.

$$|2x+18|+1=11$$ Subtract 1

$|2x+18|+1-1=11-1$ Simplify

$|2x+18|=10$ Form two equations

Possibilities:

(a) There are exactly two solutions: $x=-4$ and $x=4$.

(b) The only solution is $x=-4$.

(c) The only solution is $x=-14$.

(d) There are exactly two solutions: $x=-4$ and $x=-14$.

(e) The equation does not have any solutions.

$2x+18=10$ and $2x+18=-10$

$-18 \quad -18 \qquad\qquad -18 \quad -18$

$\frac{2x}{2}=\frac{-8}{2} \qquad\qquad \frac{2x}{2}=\frac{-28}{2}$

$x=-4 \qquad\qquad x=-14$

---

37. In the picture below, the graph of an equation involving x and y is shown. Find the true statement.

Check $x=-4$

$|2(-4)+18|+1 \stackrel{?}{=} 11$

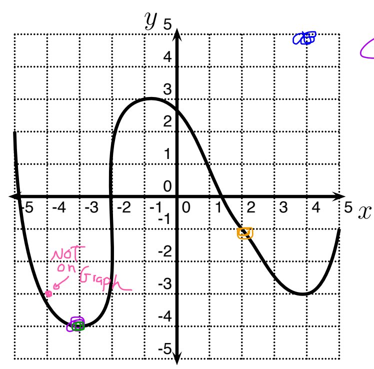
$|-8+18|+1 \stackrel{?}{=} 11$

$|10|+1 \stackrel{?}{=} 11$

$10+1 \stackrel{?}{=} 11$

$11=11$

☺ So $x=-4$ is a solution



Check $x=-14$

$|2(-14)+18|+1 \stackrel{?}{=} 11$

$|-28+18|+1 \stackrel{?}{=} 11$

$|-10|+1 \stackrel{?}{=} 11$

$10+1 \stackrel{?}{=} 11$

$11=11$

So $x=-14$ is a solution

Possibilities:

(a) $(-3,-4)$ is a solution to the equation.

(b) $(4,5)$ is a solution to the equation.

(c) $(2,-1)$ is a ~~NOT~~ solution to the equation.

(d) $(-3,-4)$ is a ~~NOT~~ solution to the equation.

(e) $(-4,-3)$ is a solution to the equation.