MA 109 — College Algebra
EXAM 3 - REVIEW

Name: ______________________ **Sec.:** ________

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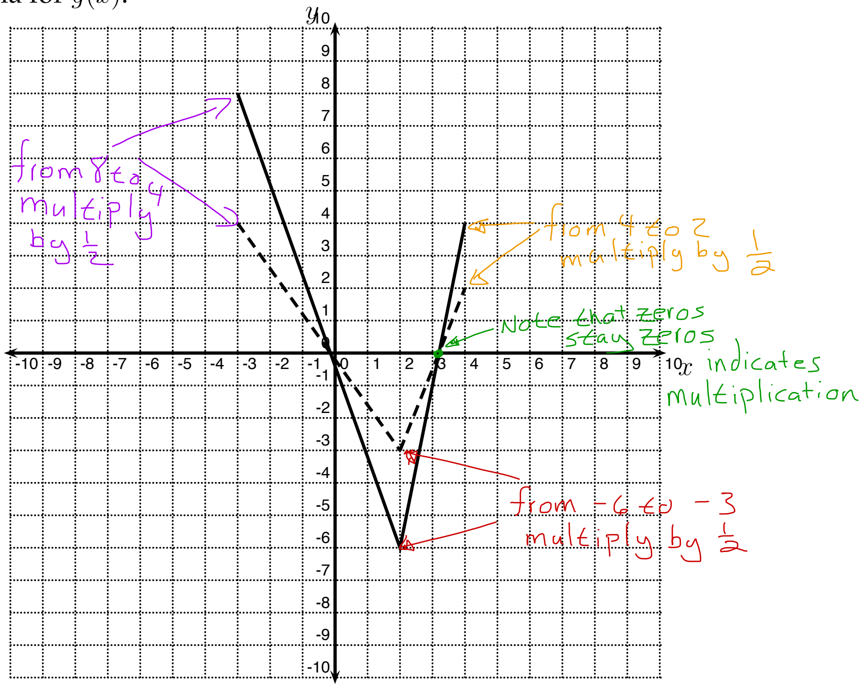
1. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$.



Possibilities:

(a) $g(x) = f(2x)$

(b) $g(x) = 2f(x)$

(c) $g(x) = f\left(\frac{1}{2}x\right)$

(d) $g(x) = -2f(x)$

(e) $\boxed{g(x) = \frac{1}{2}f(x)}$

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2. Suppose that the graph of $y = f(x)$ contains the point (5,3) $f(5)=3$. Find a point on the graph of $y =$ $g(x)=$ $2f(x) - 4$. then $g(5) = 2 \cdot f(5) - 4 = 2 \cdot 3 - 4 = 6 - 4 = 2$ so g contains the point $(5,2)$

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3. Let $f(x) = |x + 4| - 4$. Find the average rate of change of $f(x)$ between $x = \underset{a}{-8}$ and $x = \underset{b}{9}$.

Possibilities:

(a) $-9/17$

(b) 9

(c) 17/9

(d) 9/17

(e) -9

Note: $f(9) = |9+4| - 4 = |13| - 4 = 13 - 4 = 9$
and $f(-8) = |-8+4| - 4 = |-4| - 4 = 4 - 4 = 0$

$$\text{average Rate of Change} = \frac{f(b) - f(a)}{b - a} = \frac{f(9) - f(-8)}{9 - (-8)} = \frac{9 - 0}{9 + 8} = \boxed{\frac{9}{17}}$$

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4. Suppose that the graph of $y = f(x)$ contains the point (10,-20) $f(10) = -20$. Find a point on the graph of $y = f(5x) + 7$.

$g(x) =$

1st Need $5x = 10$ Divide by 5

$\frac{\cancel{5}x}{\cancel{5}} = \frac{10}{5}$ Simplify

$x = 2$

Thus, $g(2) = f(5 \cdot 2) + 7 = f(10) + 7 = -20 + 7 = -13$

Consequently, g contains the point $\boxed{(2,-13)}$

5. Let $f(x) = x^2 + 6x$. Find the average rate of change of $f(x)$ from $x = \overset{a}{a}$ to $x = \overset{b}{a+h}$. Assume $h \neq 0$.

Possibilities:

(a) $-2a - h - 6$

(b) $\dfrac{2ah + h^2 + 12a + 6h}{h}$

(c) 1

[✓ (d) $2a + h + 6$]

(e) $\dfrac{h^2 + 6h}{h}$

Note: $f(a+h) = (a+h)^2 + 6(a+h) = (a+h)(a+h) + 6a + 6h = a^2 + ah + ha + h^2 + 6a + 6h$

$= a^2 + 2ah + h^2 + 6a + 6h$

and $f(a) = a^2 + 6a$

The Average Rate of change $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{f(a+h) - f(a)}{a + h - a} = \dfrac{(a^2 + 2ah + h^2 + 6a + 6h) - (a^2 + 6a)}{h}$

$= \dfrac{\cancel{a^2} + 2ah + h^2 + \cancel{6a} + 6h \cancel{- a^2} \cancel{- 6a}}{h} = \dfrac{2ah + h^2 + 6h}{h}$

$= \dfrac{\cancel{h}(2a + h + 6)}{\cancel{h}} = \boxed{2a + h + 6}$

6. The mass $m(t)$ remaining after t years from a 80-gram sample of a radioactive elmement is given by $m(t) = 80e^{-0.2t}$. When will the mass remaining equal 8 grams? Round your answer to the nearest hundredth of a year.

Possibilities:

(a) About 0.18 years

[✓ (b) About 11.51 years]

(c) About 2.30 years

(d) About 0.12 years

(e) About 5.00 years

$8 = 80e^{-0.2t}$ Divide by 8

$\dfrac{8}{80} = \dfrac{\cancel{80}e^{-0.2t}}{\cancel{80}}$ Simplify

$\dfrac{1}{10} = e^{-0.2t}$ Apply ln

$\ln(\frac{1}{10}) = \ln(e^{-0.2t})$ $\ln(e^x) = x$

$\ln(\frac{1}{10}) = -0.2t$ Divide by -0.2

$\dfrac{\ln(\frac{1}{10})}{-0.2} = \dfrac{\cancel{-0.2}t}{\cancel{-0.2}}$ Simplify

$t = \dfrac{\ln(\frac{1}{10})}{-0.2} \approx \boxed{11.51 \text{ years}}$

Recall Properties:

① $\log_a(m) + \log_a(N) = \log_a(m \cdot N)$

② $y = \log_a(x)$ means $a^y = x$

7. Solve.

$$\log_8(x-5) + \log_8(x+2) = 1$$

Possibilities:

(a) $x_1 = 6$ and $x_2 = 3$

(b) $x_1 = 5$ and $x_2 = -2$

[✓ (c) $x_1 = 6$]

(d) $x_1 = 5$

(e) $x_1 = 6$ and $x_2 = -3$

$\log_8(x-5) + \log_8(x+2) = 1$ Property 1

$\log_8((x-5)(x+2)) = 1$ Property 2

$8^1 = (x-5)(x+2)$ Simplify (Distribute)

$8 = x^2 + 2x - 5x - 10$ Simplify

$8 = x^2 - 3x - 10$ Subtract 8

$8 - 8 = x^2 - 3x - 10 - 8$ Simplify

$0 = x^2 - 3x - 18$ Factor

$0 = x^2 + 3x - 6x - 18$

$0 = x(x+3) - 6(x+3)$

$0 = (x-6)(x+3)$ Zero Product Rule

$x - 6 = 0$
$+6 \quad +6$
$x = 6$

$x + 3 = 0$
$-3 \quad -3$
$x = -3$

Must check!! See Next Page

8. Find all real solutions or state that there are NONE.

$$8e^{x-6} = 2.$$

$8e^{x-6} = 2$ Divide by 8

$\dfrac{\cancel{8}e^{x-6}}{\cancel{8}} = \dfrac{2}{8}$ Simplify

$e^{x-6} = \dfrac{1}{4}$ Apply ln

$\ln(e^{x-6}) = \ln(\frac{1}{4})$ $\ln(e^x) = x$

$x - 6 = \ln(\frac{1}{4})$ Add 6

$x - 6 + 6 = \ln(\frac{1}{4}) + 6$ Simplify

$\boxed{x = \ln(\frac{1}{4}) + 6}$

$\boxed{\#7}$ Check $x=6$

$$\log_8(6-5)+\log_8(6+2)\overset{?}{=}1$$
$$\log_8(1)+\log_8(8)\overset{?}{=}1$$
$$0+1\overset{?}{=}1$$
$$1=1$$ ☺

So $x=6$ is a solution

Check $x=-3$

$$\log_8(-3-5)+\log_8(-3+2)\overset{?}{=}1$$
$$\log_8(-8)+\log_8(-1)\overset{?}{=}1$$

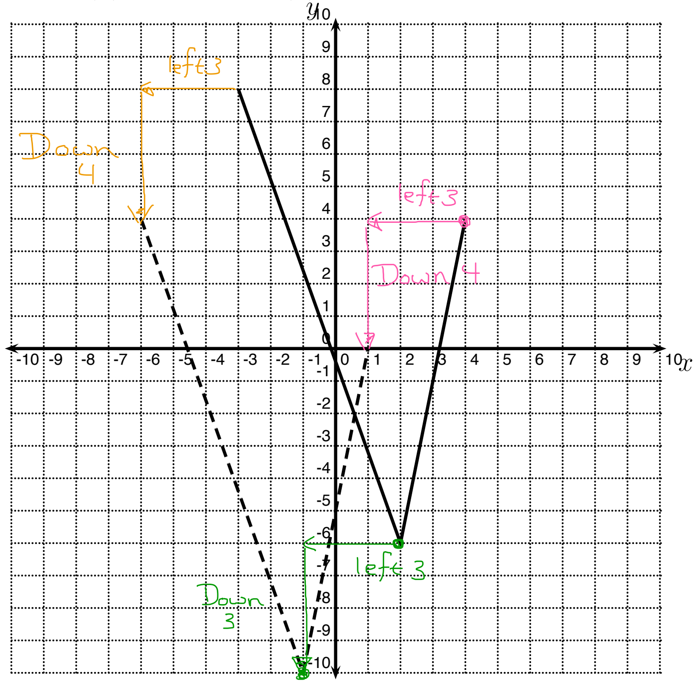
undefined ☹

Domain of $f(x)=\log_8(x)$ is $x>0$

So $x=-3$ is <u>Not</u> a solution

9. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$ in terms of $f(x)$.

Geometry	Algebra	Equation
left 3	Replace x with x+3	$f(x+3)$
Down 4	Subtract 4	$f(x+3) - 4$



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10. Suppose that the graph of $y = f(x)$ contains the point (-2,7). Find a point on the graph of $y = g(x) = 5f(x+3)$.

$f(-2) = 7$

1st Need $x+3 = -2$ Subtract 3

$x+3-3 = -2-3$ Simplify

$x = -5$

Thus, $g(-5) = 5f(-5+3) = 5 \cdot f(-2) = 5 \cdot 7 = 35$

Consequently, the graph of g contains the point $(-5, 35)$

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11. Let $f(x) = 6x + 4$. Find the average rate of change of $f(x)$ from $x = -5$ to $x = 9$.

Possibilities:

(a) 3

$\boxed{\text{(b) } 6}$

(c) 5

(d) 2

(e) 4

Note $f(9) = 6 \cdot 9 + 4 = 54 + 4 = 58$

and $f(-5) = 6 \cdot (-5) + 4 = -30 + 4 = -26$

Average Rate of Change $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{f(9) - f(-5)}{9 - (-5)} = \dfrac{58 - (-26)}{9 - (-5)} = \dfrac{58 + 26}{9 + 5} = \dfrac{84}{14} = 6$

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12. Let $f(x) = \ln(3x + 7)$. Find $f^{-1}(x)$.

$f^{-1}(x) = \dfrac{e^x - 7}{3}$ See work on next page

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13. Let $f(x) = 2e^{5x+3}$. Find $f^{-1}(x)$.

$f^{-1}(x) = \dfrac{\ln\left(\frac{x}{2}\right) - 3}{5}$ See work on next page

#12

$f(x) = \ln(3x+7)$ 1st Replace $f(x)$ with y

$y = \ln(3x+7)$ 2nd Interchange x and y

$x = \ln(3y+7)$ 3rd Solve for y

$x = \ln(3y+7)$ Recall $\ln = \log_e$

$x = \log_e(3y+7)$ $y = \log_a(x)$ means $a^y = x$

$e^x = 3y+7$ Subtract 7

$e^x - 7 = 3y+7-7$ Simplify

$e^x - 7 = 3y$ Divide by 3

$\frac{e^x-7}{3} = \frac{\cancel{3}y}{\cancel{3}}$ Simplify

$\frac{e^x-7}{3} = y$ 4th Replace y with $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \frac{e^x-7}{3}}$$

#13

$f(x) = 2e^{5x+3}$ 1st Replace $f(x)$ with y

$y = 2e^{5x+3}$ 2nd Interchange x and y

$x = 2e^{5y+3}$ 3rd Solve for y

$x = 2e^{5y+3}$ Divide by 2

$\frac{x}{2} = \frac{\cancel{2}e^{5y+3}}{\cancel{2}}$ Simplify

$x/2 = e^{5y+3}$ Apply ln

$\ln\left(\frac{x}{2}\right) = \ln\left(e^{5y+3}\right)$ $\ln(e^x) = x$

$\ln\left(\frac{x}{2}\right) = 5y+3$ Subtract 3

$\ln\left(\frac{x}{2}\right) - 3 = 5y+3-3$ Simplify

$\ln\left(\frac{x}{2}\right) - 3 = 5y$ Divide by 5

$\frac{\ln\left(\frac{x}{2}\right) - 3}{5} = \frac{\cancel{5}y}{\cancel{5}}$ Simplify

$\frac{\ln\left(\frac{x}{2}\right) - 3}{5} = y$ 4th Replace y with $f^{-1}(x)$

$$f^{-1}(x) = \frac{\ln\left(\frac{x}{2}\right) - 3}{5}$$

14. The number of bacteria in a culture is modeled by the function $n(t) = 80e^{.45t}$ where t is measured in hours. After how many hours ~~will~~ until the number of bacteria reach 7000?

Possibilities:

- (a) About 9.94 hours [✓]
- (b) About 4.47 hours
- (c) About 71.53 hours
- (d) About 4.32 hours
- (e) About 11.51 hours

$7000 = 80e^{0.45t}$ Divide by 80

$\frac{7000}{80} = \frac{\cancel{80}e^{0.45t}}{\cancel{80}}$ Simplify

$87.5 = e^{0.45t}$ Apply ln

$\ln(87.5) = \ln(e^{0.45t})$ $\ln(e^x) = x$

$\ln(87.5) = 0.45t$ Divide by 0.45

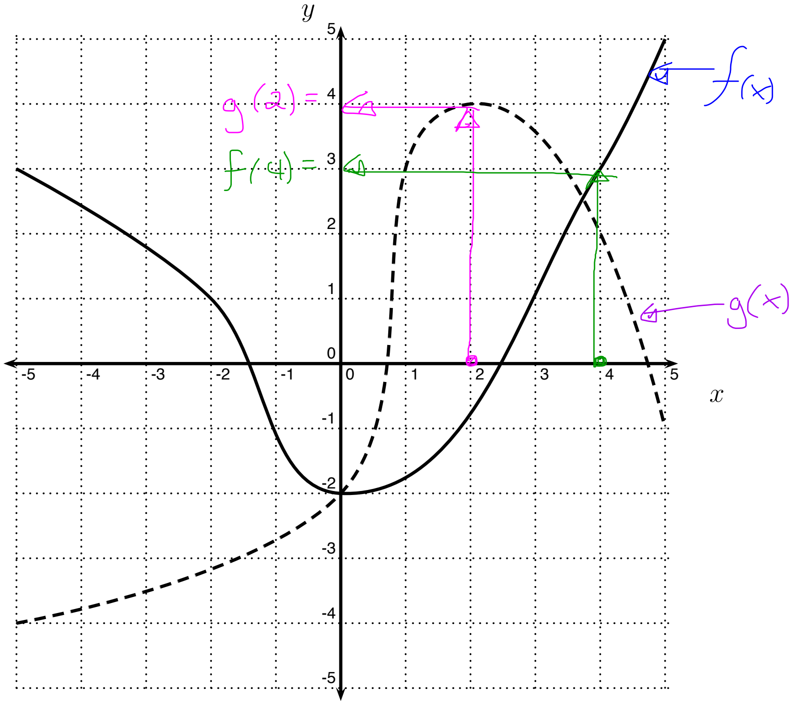
$\frac{\ln(87.5)}{0.45} = \frac{\cancel{0.45}t}{\cancel{0.45}}$ Simplify

$t = \frac{\ln(87.5)}{0.45}$

$t \approx 9.94$ hours

15. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Use the graphs to evaluate $f(g(2))$.

$f(g(2)) = f(4) = 3$



16. Let $f(x) = \sqrt{x-3}$ and $g(x) = \sqrt{4-x}$. Find the domain of $(f+g)(x)$. $= f(x) + g(x)$

$= \sqrt{x-3} + \sqrt{4-x}$

17. Let $f(x) = 3x - 7$. Find $f(f(x))$ $= f(3x-7) = 3(3x-7) - 7$

$= 9x - 21 - 7$

$= \boxed{9x - 28}$

P_0 $r=0.08$ $n=4$

18. If $5000 is invested at an interest rate of 8% per year compounded quarterly, find the amount of the investment at the end of 18 years.

$t=18$

Possibilities:

(a) $6802.44

(b) $5412.16

(c) $20805.70

(d) $7141.23

(e) $1274912.56

Recall Compound Interest Formula

$$P(t)=P_0\left(1+\frac{r}{n}\right)^{nt}$$

$$P(18)=5000\left(1+\frac{0.08}{4}\right)^{4\cdot 18}$$

$$=5000(1.02)^{72}$$

$$=20805.70$$

15% discount means you pay 100%-15%=85% of the value of the item.

$$\underbrace{\overbrace{(\underset{\text{original price}}{175}-\underset{\text{-coupon}}{10})}^{\text{Price after coupon}}*0.85}_{\text{Price after Coupon and then 15\% discount}}=140.25$$

19. You wish to purchase a new cell phone. You have a coupon for $10 and the store is running a special which allows you to deduct 15% from the price. If the original price of the cell phone is 175 dollars, what is the final price if you apply the coupon and then apply the 15% discount.

$\boxed{\$140.25}$

20. Let $h(x) = (2x + 1)^5$. Find functions $f(x)$ and $g(x)$ such that $h(x) = f(g(x))$.

Possibilities:

(a) $f(x) = (2x + 1)^3$ and $g(x) = (2x + 1)^2$

(b) $f(x) = x^5$ and $g(x) = 2x + 1$

(c) $f(x) = x^5 + 1$ and $g(x) = 2x$

(d) $f(x) = 2x$ and $g(x) = x^5 + 1$

(e) $f(x) = 2x + 1$ and $g(x) = x^5$

Let $g(x)=2x+1$

and $f(x) = x^5$

then $f(g(x))=f(2x+1)=(2x+1)^5=h(x)$

21. Which of the following functions are one-to-one?

~~(a)~~. $a(x) = |x|$

(b). $b(x) = x^3$

(c). $c(x) = x^3 + 1$

~~(d)~~. $d(x) = 5$

~~(e)~~. $e(x) = x^4$

~~(f)~~. $f(x) = x^4 + 1$

(g). $g(x) = (x + 1)^3$

~~(h)~~. $h(x) = |x + 1|$

(i). $i(x) = 2x + 3$

(j). $j(x) = \sqrt{x}$

(k). $k(x) = \sqrt[3]{x}$

~~(l)~~. $l(x) = 0$

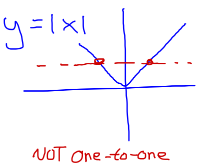
shifted up one unit

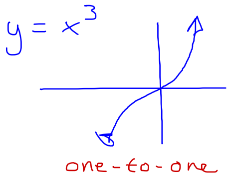
shifted up one unit

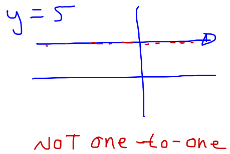
shifted left one unit

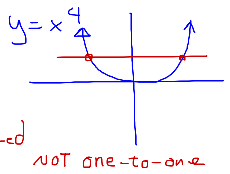
shifted left one unit

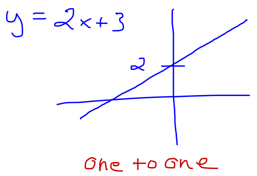
Similar to "d"

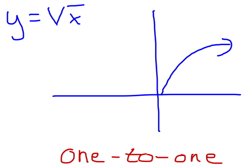


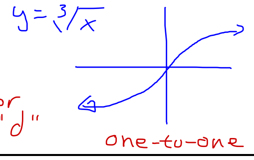












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22. Let $f(x) = \sqrt{x-1}$ and $g(x) = x^2 - 4$. Find the domain of $\left(\frac{g}{f}\right)(x)$. $(1, \infty)$ See work on Next page

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23. Let $f(x) = \sqrt{x-1}$ and $g(x) = x^2 - 4$. Find the domain of $\left(\frac{f}{g}\right)(x)$. $[1,2) \cup (2,\infty)$ See work on Next Page

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24. Let $f(x) = \frac{2x+3}{4-5x}$. Find $f^{-1}(x)$. $= \frac{3-4x}{-5x-2}$ See work on following page

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25. Let $f(x) = 2x^2 + 7x$. Find $\frac{f(x+h)-f(x)}{h}$.

Possibilities:

1st Note: $f(x+h) = 2(x+h)^2 + 7(x+h) = 2(x+h)(x+h) + 7x + 7h$

$= 2(x^2 + xh + hx + h^2) + 7x + 7h = 2(x^2 + 2xh + h^2) + 7x + 7h$

$= 2x^2 + 4xh + 2h^2 + 7x + 7h$

(a) $-4x - 2h - 7$

(b) $\frac{2h^2+7h}{h}$

(c) $2h^2 + 7h$

(d) $\boxed{4x + 2h + 7}$

(e) $\frac{4xh + 2h^2 + 14x + 7h}{h}$

Consequently, $\frac{f(x+h) - f(x)}{h} = \frac{(2x^2 + 4xh + 2h^2 + 7x + 7h) - (2x^2 + 7x)}{h}$

$= \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+7x} + 7h \cancel{-2x^2} \cancel{-7x}}{h} = \frac{4xh + 2h^2 + 7h}{h}$

$= \frac{\cancel{h}(4x + 2h + 7)}{\cancel{h}} = 4x + 2h + 7$

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26. $\log\left(\frac{x^{-2}}{y^5 z^8}\right) \overset{②}{=} \log(x^{-2}) - \log(y^5 \cdot z^8) \overset{①}{=} \log(x^{-2}) - [\log(y^5) + \log(z^8)]$

Possibilities:

Distribute → $= \log(x^{-2}) - \log(y^5) - \log(z^8)$

③ → $= -2\log(x) - 5\log(y) - 8\log(z)$

(a) $-2\log(x)/(5\log(y) * 8\log(z))$

(b) $-2\log(x) - 5\log(y) + 8\log(z)$

(c) $-2\log(x) + 5\log(y) - 8\log(z)$

(d) $\boxed{-2\log(x) - 5\log(y) - 8\log(z)}$

(e) $-2\log(x)/5\log(y) * 8\log(z)$

Properties: ① $\log_a(m * N) = \log_a(m) + \log_a(N)$

② $\log_a\left(\frac{m}{N}\right) = \log_a(m) - \log_a(N)$

③ $\log_a(m^r) = r\log_a(m)$

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27. Let $f(x) = \log_2(x+7) - 3$. Find $f^{-1}(x)$. $= 2^{x+3} - 7$

1st Replace f(x) with y $\quad y = \log_2(x+7) - 3$

2nd Interchange x and y $\quad x = \log_2(y+7) - 3$

3rd Solve for y

$x = \log_2(y+7) - 3$ Add 3

$x + 3 = \log_2(y+7) - 3 + 3$ Simplify

$x + 3 = \log_2(y+7)$

$y = \log_a(x)$ means $a^y = x$

$2^{x+3} = y + 7$ Subtract 7

$2^{x+3} - 7 = y + 7 - 7$ Simplify

$2^{x+3} - 7 = y$

4th Replace y with $f^{-1}(x)$

$$\boxed{f^{-1}(x) = 2^{x+3} - 7}$$

$\boxed{\#22}$ Domain of $f(x)=\sqrt{x-1}$

must have $\quad x-1\geq 0 \quad$ Add one (same)

$x-1+1\geq 0+1 \quad$ Simplify (same)

$x\geq 1$

Domain of $g(x)=x^2-4$ polynomial so domain is all Real numbers

Region in common is $x\geq 1$

Must find out when $f(x)=0$

$\sqrt[2]{x-1}=0 \quad \sqrt[n]{a}=b$ means $b^n=a$

$0^2=x-1 \quad$ Simplify

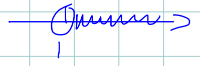
$0=x-1 \quad$ Add one

$0+1=x-1+1 \quad$ Simplify

$1=x$

$x=1$ must be removed from the common region

Consequently, the domain of $\left(\frac{g}{f}\right)(x)$ is $x>1$ or



or

$(1,\infty)$

$\boxed{\#23}$ Same Common region as above $x\geq 1$

However now one must find out when $g(x)=0$

$x^2-4=0 \quad$ Factor (Difference of Squares)

$(x-2)(x+2)=0 \quad$ Zero Product Rule

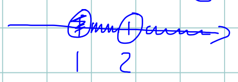
$x-2=0 \qquad x+2=0$

$+2 \quad +2 \qquad -2 \quad -2$

$x=2 \qquad x=-2$

So $x=2$ and $x=-2$ must be removed from the common region

Consequently, the domain of $\left(\frac{f}{g}\right)(x)$ is



or

$[1,2)\cup(2,\infty)$

$\boxed{\#24}$

$f(x) = \frac{2x+3}{4-5x}$ 1st Replace $f(x)$ with y

$y = \frac{2x+3}{4-5x}$ 2nd Interchange x and y

$x = \frac{2y+3}{4-5y}$ 3rd Solve for y

$x = \frac{2y+3}{4-5y}$ Multiply by $4-5y$

$(4-5y)x = \left(\frac{2y+3}{\cancel{4-5y}}\right)\cancel{(4-5y)}$ Simplify

$4x - 5xy = 2y + 3$ Subtract $2y$ ← All terms with a "y" move to left and everything else on right.

$4x - 5xy - 2y = 2y + 3 - 2y$ Simplify

$4x - 5xy - 2y = 3$ Subtract $4x$

$4x - 5xy - 2y - 4x = 3 - 4x$ Simplify

$-5xy - 2y = 3 - 4x$ Factor out a "y"

$y(-5x-2) = 3 - 4x$ Divide by $-5x-2y$

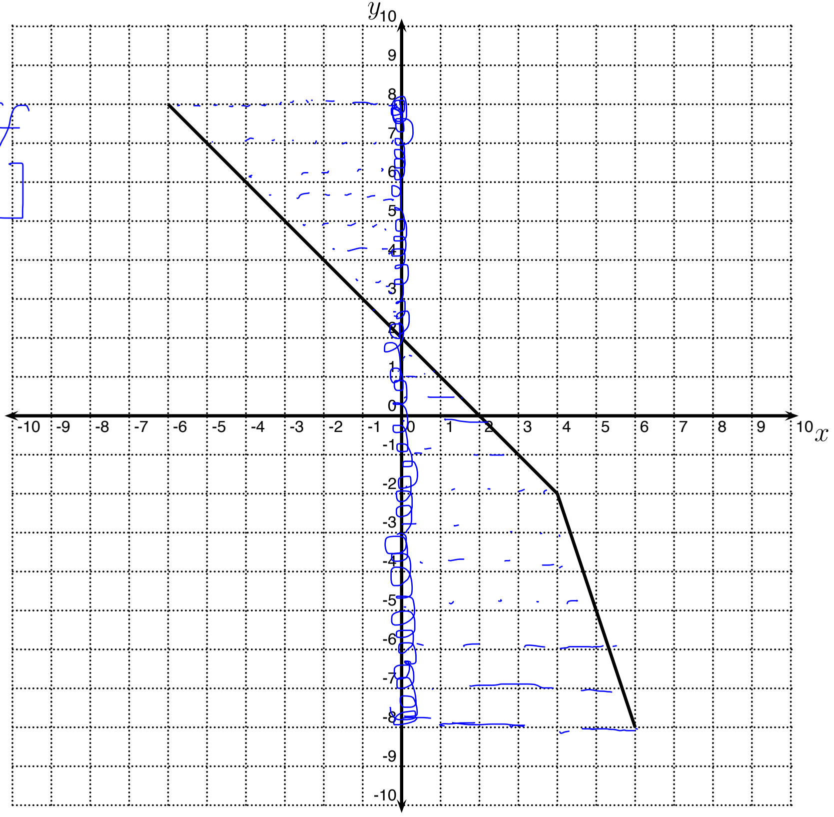
$\frac{y\cancel{(-5x-2)}}{\cancel{-5x-2}} = \frac{3-4x}{-5x-2}$ Simplify

$y = \frac{3-4x}{-5x-2}$ Replace y with $f^{-1}(x)$

$f^{-1}(x) = \frac{3-4x}{-5x-2}$

28. The graph of the one-to-one function f is shown below. Find the domain of f^{-1}.

Domain of f^{-1} = Range of f
$= [-8, 8]$



29. Joni invests \$4000 at an interest rate of 4% per year compounded continuously. How much time will it take for the value of the investment to quadruple? Round your answer to the nearest tenth of a year.

P

$r = 0.04$

A

$A = Pe^{rt}$

Possibilities:

(a) 17.3 years

(b) 27.5 years

(c) 16.0 years

$\boxed{\text{(d) 34.7 years}}$

(e) 40.2 years

$A = 4 \cdot 4000$
$= 16000$

$A = Pe^{rt}$

$16000 = 4000e^{0.04t}$ Divide by 4000

$\frac{16000}{4000} = \frac{\cancel{4000}e^{0.04t}}{\cancel{4000}}$ Simplify

$4 = e^{0.04t}$ Apply ln

$\ln(4) = \ln(e^{0.04t})$ $\ln(e^x) = x$

$\ln(4) = 0.04t$ Divide by 0.04

$\frac{\ln(4)}{0.04} = \frac{\cancel{0.04}t}{\cancel{0.04}}$ Simplify

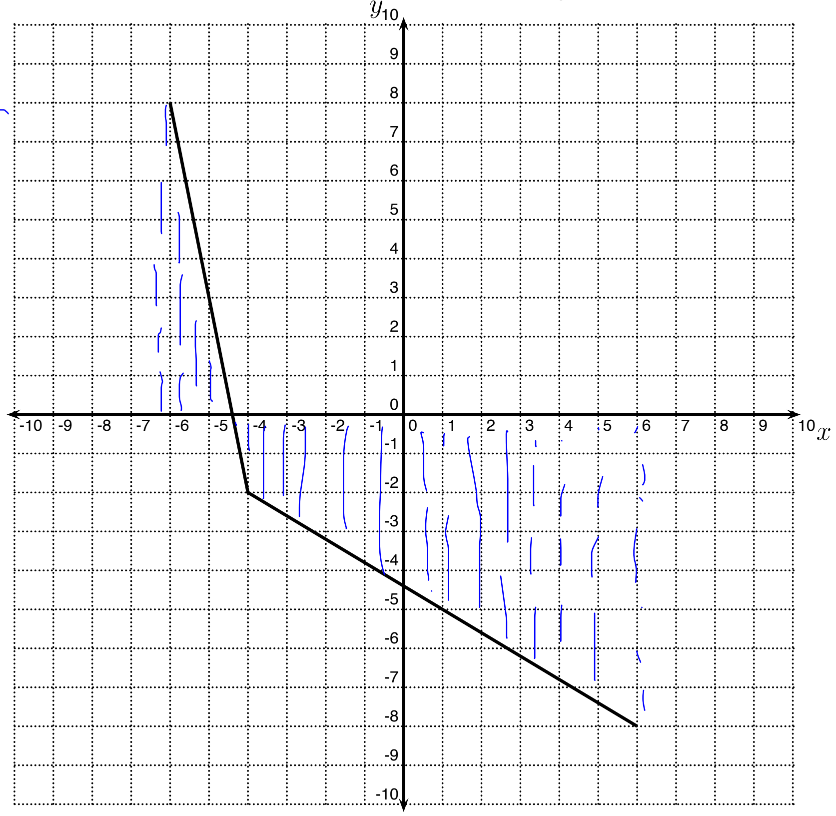
$t = \frac{\ln(4)}{0.04}$

$t \approx 34.7$ years

30. The graph of the one-to-one function f is shown below. Find the range of f^{-1}.

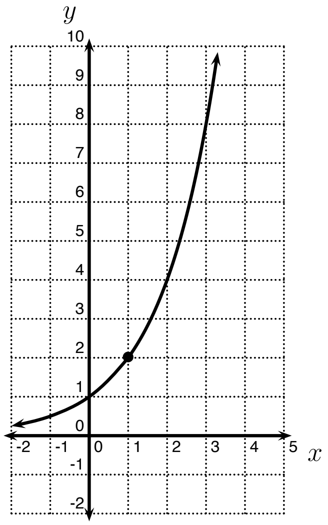
Range of f^{-1} = Domain of f

$= [-6, 6]$



---

31. The graph of an exponential function, $f(x) = a^x$ is shown below. Find a.



Note $f(1) = 2$ by the graph below. However

$f(1) = a^1 = a$

so $a = f(1) = 2$

that is, $a = 2$

Possibilities:

(a) 4

(b) $\frac{1}{2}$

(c) 3

(d) $\frac{1}{3}$

$\boxed{\text{(e) } 2}$

---

32. Solve.

$$\log_8(x-5)+\log_8(x+2)=1$$

Possibilities:

[✓ (a) $x_1 = 6$]
(b) $x_1 = 5$ and $x_2 = -2$
(c) $x_1 = 5$
(d) $x_1 = 6$ and $x_2 = 3$
(e) $x_1 = 6$ and $x_2 = -3$

See work from problem #7

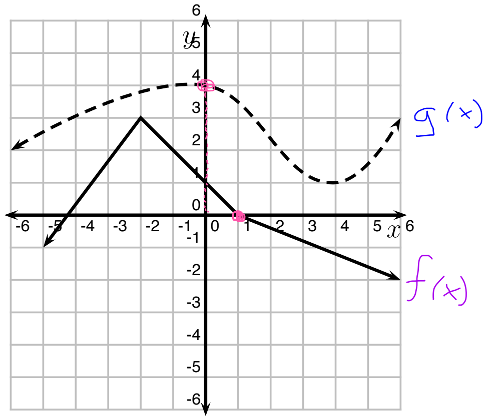
---

33. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Use the graphs to evaluate $g(f(1))$.

$g(f(1)) = g(0) = 4$

Possibilities:

(a) -1
(b) 0
(c) -2
[✓ (d) 4]
(e) 3



34. Let $f(x) = \log_4(-15 - x)$. Find the domain of $f(x)$.

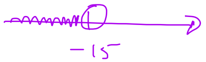
Possibilities:

(a) $(-\infty, -15]$
(b) $(-15, 15)$
(c) $[-15, 4)$
(d) $(4, \infty)$
[✓ (e) $(-\infty, -15)$]

$-15 - x > 0$ Add x (same)

$-15 - x + x > 0 + x$ Simplify (same)

$-15 > x$



$(-\infty, -15)$

35. Find the domain and range of $y = \ln(x-7)$.

Possibilities:

(a) Domain: $(-\infty,\infty)$ Range: $[7,\infty)$

(b) Domain: $(-\infty,\infty)$ Range: $(7,\infty)$

(c) Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$

(d) Domain: $[7,\infty)$ Range: $(-\infty,\infty)$

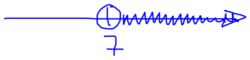
[✓ (e) Domain: $(7,\infty)$ Range: $(-\infty,\infty)$]

Domain:

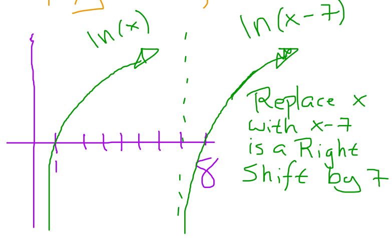
$x - 7 > 0$ Add 7 (same)

$x - 7 + 7 > 0 + 7$ Simplify (same)

$x > 7$



$(7,\infty)$



Replace x with x-7 is a Right Shift by 7

Range = $(-\infty,\infty)$

36.

$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$

$$\log_5\left(\frac{x^3\sqrt{z}}{y^2}\right) = \log_5(x^3\sqrt{z}) - \log_5(y^2)$$

$\log_a(m \cdot n) = \log_a(m) + \log_a(n) \longrightarrow = \log_5(x^3) + \log_5(\sqrt{z}) - \log_5(y^2)$

$\sqrt{a} = a^{\frac{1}{2}} \rightarrow = \log_5(x^3) + \log_5(z^{\frac{1}{2}}) - \log_5(y^2)$

$\log_a(m^r) = r\log_a(m) \rightarrow = 3\log_5(x) + \frac{1}{2}\log_5(z) - 2\log_5(y)$

Possibilities:

[✓ (a) $3\log_5(x) + \frac{1}{2}\log_5(z) - 2\log_5(y)$]

(b) $\dfrac{(3\log_5(x))\left(\frac{1}{2}\log_5(z)\right)}{2\log_5(y)}$

(c) $3x + \frac{1}{2}z - 2y$

(d) $2\log_5(y) - 3\log_5(x) - \frac{1}{2}\log_5(z)$

(e) $\dfrac{(3x)\left(\frac{1}{2}z\right)}{2y}$

---

37. Solve for x.

$8\log(x+5) = 16$ Divide by 8

$\frac{\cancel{8}\log(x+5)}{\cancel{8}} = \frac{16}{8}$ Simplify

$\log(x+5) = 2$ $\log := \log_{10}$

$\log_{10}(x+5) = 2$ $y = \log_a(x)$ means $a^y = x$

$10^2 = x + 5$ Subtract 5

$10^2 - 5 = x + 5 - 5$ Simplify

$10^2 - 5 = x$

$\boxed{x = 10^2 - 5}$

Possibilities:

(a) $x = \dfrac{16}{8 - \log(5)}$

[✓ (b) $x = 10^2 - 5$]

(c) $x = \dfrac{10^{16}}{8} - 5$

(d) $x = \dfrac{10^{16} - 5}{8}$

(e) $x = \dfrac{16}{8\log(5)}$

---

38. Let $f(x) = 2x + 1$. Find $f^{-1}(5)$.

Possibilities:

- (a) 12
- (b) 1
- (c) 11
- (d) 3
- (e) 2

$y = f(x)$ means $f^{-1}(y) = x$

so $5 = f(x)$ means $f^{-1}(5) = x$ ← Note we are looking for "x"

However, $f(x) = 2x+1$ so we have

$5 = f(x) = 2x+1$

$5 = 2x + 1$ Subtract 1

$5 - 1 = 2x + 1 - 1$ Simplify

$4 = 2x$ Divide by 2

$\frac{4}{2} = \frac{\cancel{2}x}{\cancel{2}}$ Simplify

$2 = x$ Therefore $f^{-1}(5) = 2$

---

39. Let $f(x) = 2x - 5$ and $g(x) = x^2$. Find $g(f(x))$.

Possibilities:

- [✓ (a) $4x^2 - 20x + 25$]
- (b) $2x^2 - 5$
- (c) $4x^2 - 25$
- (d) $2x^3 - 5x^2$
- (e) $4x^2 + 25$

$g(f(x)) = g(2x-5) = (2x-5)^2 = (2x-5)(2x-5)$

$= 4x^2 - 10x - 10x + 25$

$= 4x^2 - 20x + 25$

---

40. Let $f(x) = 3x + 4$. Find the average rate of change of $f(x)$ from $x \underset{\uparrow a}{=} -5$ to $x \underset{\uparrow b}{=} 9$.

Possibilities:

- (a) 5
- [✓ (b) 3]
- (c) 6
- (d) 2
- (e) 4

Note: $f(9) = 3 \cdot 9 + 4 = 27 + 4 = 31$

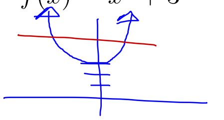
and $f(-5) = 3 \cdot (-5) + 4 = -15 + 4 = -11$

The average Rate of Change $= \frac{f(b) - f(a)}{b - a} = \frac{f(9) - f(-5)}{9 - (-5)} = \frac{31 - (-11)}{9 - (-5)} = \frac{31+11}{9+5} = \frac{42}{14} = \boxed{3}$

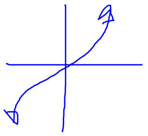
---

41. Which of the following functions are one-to-one?

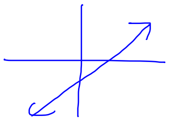
$f(x) = x^2 + 3$ $\qquad g(x) = x^3$ $\qquad h(x) = 3x - 9$



NOT ONE-TO-ONE



one-to-one



one-to-one

Possibilities:

(a) Only $f(x)$ and $g(x)$ are one-to-one.

[✓ (b) Only $g(x)$ and $h(x)$ are one-to-one.]

(c) Only $h(x)$ is one-to-one.

(d) None of the functions are one-to-one.

(e) All of the functions are one-to-one.

---

42. Use a calculator to approximate $\log_{13}(15)$. Your answer should be correct to 5 decimal places.

Possibilities:

(a) 0.14310

[✓ (b) 1.05579]

(c) 0.94716

(d) 1.15385

(e) 1.76611

$$\log_{13}(15) = \frac{\log(15)}{\log(13)} = 1.05579$$

---

43. Let $f(x) = 3x^2 + 2x + 1$. Find $\dfrac{f(x+h) - f(x)}{h}$

1st Compute $f(x+h) = 3(x+h)^2 + 2(x+h) + 1$

$$= 3(x+h)(x+h) + 2x + 2h + 1$$
$$= 3(x^2 + xh + hx + h^2) + 2x + 2h + 1$$
$$= 3(x^2 + 2xh + h^2) + 2x + 2h + 1$$
$$= 3x^2 + 6xh + 3h^2 + 2x + 2h + 1$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(3x^2+6xh+3h^2+2x+2h+1)-(3x^2+2x+1)}{h} = \frac{\cancel{3x^2}+6xh+3h^2\cancel{+2x}+2h\cancel{+1}\cancel{-3x^2}\cancel{-2x}\cancel{-1}}{h}$$

$$= \frac{6xh+3h^2+2h}{h} = \frac{\cancel{h}(6x+3h+2)}{\cancel{h}} = \boxed{6x+3h+2}$$

Polynomial
Domain is
all real numbers

44. Let $f(x) = \sqrt{x+5}$ and $g(x) = x - 4$. Find the domain of $\left(\frac{f}{g}\right)(x)$.

$x+5 \geq 0$ Subtract 5 (same)

Possibilities: $x+5-5 \geq 0 - 5$ Simplify (same)

$x \geq -5$

(a) $(-4, 5]$

(b) $(-\infty, -4) \cup (-4, 5]$

(c) $[-5, \infty)$

(d) $[-5, 4)$

[✓ (e) $[-5, 4) \cup (4, \infty)$]

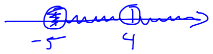
The region in common between the domain of f ($x \geq -5$)
and the domain of g (all reals) is $x \geq -5$

However we must remove value from the common region value of
"x" such that $g(x) = 0$

$x - 4 = 0$ Add 4

$x - 4 + 4 = 0 + 4$ Simplify

$x = 4$ ← so $x = 4$ must be removed



$[-5, 4) \cup (4, \infty)$

45. You are going to purchase some memory for your computer. The original price of the memory is x dollars You have two coupons. The first coupon allows you to take 20% off of the price. The second coupon allows you to deduct \$10 from the price. Suppose that you use the first coupon to take 20% off and then you use the second coupon to deduct \$10 from the price. Find a formula for the final price P in terms of x.

Possibilities:

(a) $P(x) = 0.20x - 2$

[✓ (b) $P(x) = 0.80x - 10$]

(c) $P(x) = 0.80x - 8$

(d) $P(x) = 0.80x - 8$

(e) $P(x) = 0.20x - 10$

item's price

subtract/deduct \$10 from the reduced price.

$$(\underbrace{0.80\,x}_{\text{80\% of the original value is 20\% off}}) - 10$$

46. Find the inverse function of $f(x) = 5x - 3$.

1st Replace f(x) with y $y = 5x - 3$

2nd Interchange x and y $x = 5y - 3$

3rd Solve for y

$x = 5y - 3$ Add 3

$x + 3 = 5y - 3 + 3$ Simplify

$x + 3 = 5y$ Divide by 5

$\frac{x+3}{5} = \frac{\cancel{5}y}{\cancel{5}}$ Simplify

$\frac{x+3}{5} = y$

4th Replace y with $f^{-1}(x)$ $\boxed{f^{-1}(x) = \frac{x+3}{5}}$

47. Explain how the graph of $g(x) = (x+1)^3 - 9$ is obtained from the graph of $f(x) = x^3$.

Possibilities:

(a) Shift left 9 units and shift down 1 units.
(b) Shift right 1 units and shift up 9 units.
(c) Shift right 1 units and shift down 9 units.
(d) Shift right 9 units and shift up 1 units.
[✓ (e) Shift left 1 units and shift down 9 units.]

Algebra	New function	Geometry
Replace x with x+1	$f(x+1) = (x+1)^3$	Shift Left one unit
Subtract 9	$f(x+1) - 9 = (x+1)^3 - 9$	Shift Down 9 units

NOTE: This is $g(x)$

---

48. Express the equation in logarithmic form.

$$8^3 = 512$$

Recall $y = \log_a(x)$ means $a^y = x$

$\log_8(512) = 3$

Possibilities:

(a) $\log_{512} 3 = 8$
(b) $\log_3 8 = 512$
(c) $\log_3 512 = 8$
(d) $\log_8 3 = 512$
[✓ (e) $\log_8 512 = 3$]

---

49. Which of the following statements are true?

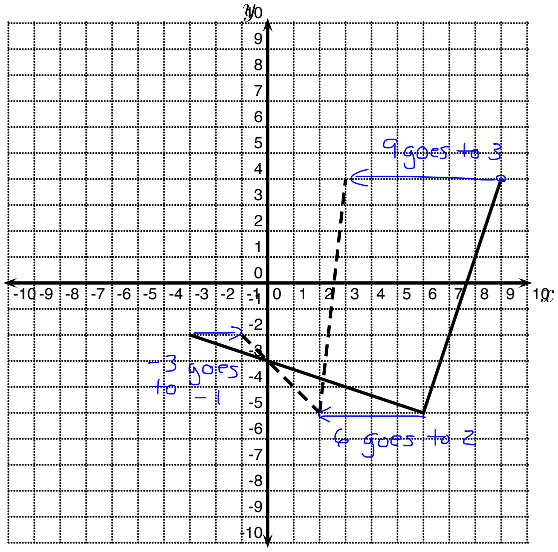
✓ (I) $2^{\log_2(5)} = 5$ $\quad a^{\log_a(x)} = x$
✓ (II) $\log(\frac{a}{b}) = \log(a) - \log(b)$ for all positive a and b.
✗ (III) $\ln(a+b) = (\ln(a))(\ln(b))$ for all positive a and b.

Counter Example
$\ln(2+5) = \ln(7) \approx 1.945910\ldots$
$\neq$
$\ln(2) \cdot \ln(5) \approx 1.115577\ldots$

Possibilities:

[✓ (a) Only (I) and (II) are true.]
(b) Only (III) is true.
(c) Only (I) and (III) are true.
(d) Only (I) is true.
(e) Statements (I), (II), and (III) are all true.

50. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$.



Scaled horizontally by a factor of $\frac{1}{3}$.
Algebraically this is a replacement of x with $3x$.

Possibilities:

(a) $g(x) = f\left(\frac{1}{3}x\right)$

[✓ (b) $g(x) = f(3x)$]

(c) $g(x) = \frac{1}{3}f(x)$

(d) $g(x) = -3f(x)$

(e) $g(x) = 3f(x)$

---

51. Let $f(x) = \log_4(8 - x)$. Find the domain of $f(x)$.

Possibilities:

(a) $(4, \infty)$

(b) $[4, 8)$

(c) $(-8, 8)$

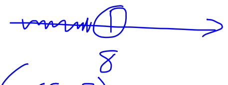
(d) $(-\infty, 8]$

[✓ (e) $(-\infty, 8)$]

$8 - x > 0$ Add x (same)

$8 - x + x > 0 + x$ Simplify (same)

$8 > x$



$(-\infty, 8)$

---

52. Use a calculator to approximate $\log_{13}(12)$. Your answer should be correct to 5 decimal places.

Possibilities:

(a) 0.92308

(b) 1.03221

(c) 1.54297

(d) -0.08004

[✓ (e) 0.96879]

$\log_{13}(12) = \frac{\ln(12)}{\ln(13)} = 0.96879\ldots\ldots$