MA 109 — College Algebra
EXAM 4 - REVIEW

Name: ______________________ **Sec.:** ________

1. Find the vertex of $y = 4x^2 - 40x + 102$. $(5,2)$ See work on next page

2. Let $f(x) = |x+4| - 6$. Find the average rate of change of $f(x)$ between $x = -8$ and $x = 9$.

   a b

   Possibilities:

   (a) $17/9$ 1^{st} Compute $f(9) = |9+4| - 6 = |13| - 6 = 13 - 6 = 7$

   (b) -9 and $f(-8) = |-8+4| - 6 = |-4| - 6 = 4 - 6 = -2$

   $\boxed{\text{(c) } 9/17}$

   (d) $-9/17$

   (e) 9

   The Average rate of change $= \dfrac{f(b)-f(a)}{b-a} = \dfrac{f(9)-f(-8)}{9-(-8)} = \dfrac{7-(-2)}{9-(-8)} = \dfrac{7+2}{9+8} = \boxed{\dfrac{9}{17}}$

3. Find an equation for the line that is perpendicular to $y = \frac{6}{7}x + 5$ and contains the point (0,13).

   $m = 6/7$ so $\perp m = -7/6$ y-intercept? $b = 13$

   Slope-intercept form $y = mx + b$

   $\boxed{y = -7/6\, x + 13}$

4. Solve.

$$\log_8(x-5) + \log_8(x+2) = 1$$

   Possibilities:

   (a) $x_1 = 6$ and $x_2 = 3$

   (b) $x_1 = 6$ and $x_2 = -3$

   (c) $x_1 = 5$

   (d) $x_1 = 5$ and $x_2 = -2$

   $\boxed{\text{(e) } x_1 = 6}$ See Review #3 problem #7

5. Suppose you want to graph $x^3 - 3y = x^2$ on your graphing calculator. What should you enter into your calculator?

   $y_1 = (x^2 - x^3)/(-3)$

6. Suppose you want to graph $(y-1)(x+14) = x$ on your graphing calculator. What should you enter into your calculator?

   $y_1 = (x/(x+14)) + 1$

$x^3 - 3y = x^2$ Subtract x^3

$x^3 - 3y - x^3 = x^2 - x^3$ Simplify

$-3y = x^2 - x^3$ Divide by -3

$\dfrac{\cancel{-3}y}{\cancel{-3}} = \dfrac{x^2 - x^3}{-3}$ Simplify

$y = \dfrac{x^2 - x^3}{-3}$

$(y-1)(x+14) = x$ Divide by $x+14$

$\dfrac{(y-1)\cancel{(x+14)}}{\cancel{x+14}} = \dfrac{x}{x+14}$ Simplify

$y - 1 = \dfrac{x}{x+14}$ Add 1

$y - 1 + 1 = \dfrac{x}{x+14} + 1$ Simplify

$y = \dfrac{x}{x+14} + 1$

$\boxed{\#1}$ $f(x) = 4x^2 - 40x + 102$

1^{st} Compute $\dfrac{-b}{2a} = \dfrac{-(-40)}{2(4)} = \dfrac{40}{8} = 5$

2^{nd} Compute $f\left(\frac{-b}{2a}\right) = f(5) = 4 \cdot 5^2 - 40 \cdot 5 + 102 = 4 \cdot 25 - 200 + 102$
$= 100 - 200 + 102 = -100 + 102 = 2$

Consequently, Vertex $= \left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right) = (5, 2)$

7. Find the vertex of $y = 4x^2 - 40x + 102$. $(5,2)$ See Problem #1

8. Which of the following statements are true?

(I) The graph of $P(x) = x^3 - 4x^2 + x + 6$ has an x-intercept at $(1, 0)$ ×

(II) The graph of $P(x) = x^3 - 4x^2 + x + 6$ has an x-intercept at $(-1, 0)$ ✓

(III) $(x - 1)$ is a factor of $P(x) = x^3 - 4x^2 + x + 6$. ×

(IV) $(x + 1)$ is a factor of $P(x) = x^3 - 4x^2 + x + 6$. ✓

Possibilities:

(a) Only (II) and (III) are true.

$\boxed{\text{(b) Only (II) and (IV) are true.}}$

(c) Only (I) and (IV) are true.

(d) Only (I) and (III) are true.

(e) None of the statements are true.

Note: $P(1) = 1^3 - 4(1)^2 + 1 + 6 = 1 - 4 + 1 + 6 = -3 + 1 + 6 = -2 + 6 = 4$

So $(1,0)$ is not an x-intercept and $(x-1)$ is not a factor of $P(x)$

$P(-1) = (-1)^3 - 4(-1)^2 + (-1) + 6 = -1 - 4 - 1 + 6 = -5 - 1 + 6 = -6 + 6 = 0$

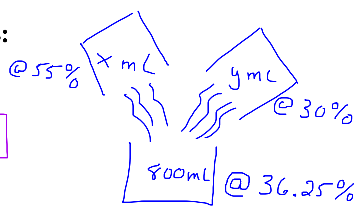
So $(-1,0)$ is an x-intercept and $(x-(-1)) = (x+1)$ is a factor of $P(x)$

9. Does the graph of $y = 3x^2 + 12x - 5$ have a maximum or a minimum? What is it? minimum

Parabola open upward since $a = 3 > 0$ therefore the graph has a minimum

10. What quantity, x, of a 55% acid solution must be mixed with a 30% acid solution to produce 800 mL of a 36.25% solution?

Possibilities:



(a) 300 mL

$\boxed{\text{(b) 200 mL}}$

(c) 700 mL

(d) 500 mL

(e) 600 mL

Total amounts: $x + y = 800$ Multiply by -0.30

Acid amounts: $0.55x + 0.30y = (0.3625)(800)$

Elimination method

$$-0.30x - 0.30y = -240$$
$$0.55x + 0.30y = 290$$
$$0.25x + 0y = 50$$
$$\frac{0.25x}{0.25} = \frac{50}{0.25}$$

$\boxed{x = 200 \text{ mL}}$

11. Find all real solutions or state that there are NONE.

$$\sqrt{x - 8} = x + 7.$$

NONE

See work on next page

12. Find all real solutions or state that there are NONE.

$$9e^{x-8} = 2.$$

$x = \ln(2/9) + 8$

See work on next page

$\boxed{\#11}$

$\sqrt[2]{x-8} = x+7$ $\quad$ $\sqrt[n]{a} = b$ means $b^n = a$

$(x+7)^2 = x-8$ $\quad$ Simplify $a^2 = a \cdot a$ & FOIL

$(x+7)(x+7) = x-8$

$x^2 + 7x + 7x + 49 = x - 8$

$x^2 + 14x + 49 = x - 8$ $\quad$ Subtract x

$x^2 + 14x + 49 - x = x - 8 - x$ $\quad$ Simplify

$x^2 + 13x + 49 = -8$ $\quad$ Add 8

$x^2 + 13x + 49 + 8 = -8 + 8$ $\quad$ Simplify

$x^2 + 13x + 57 = 0$

Note discriminant $= b^2 - 4ac = 13^2 - 4(1)(57) = 169 - 228 = -59 < 0$

Since the discriminant is less than zero there are no real solutions.

$\boxed{\#12}$

$9e^{x-8} = 2$ $\quad$ Divide by 9

$\frac{\cancel{9}e^{x-8}}{\cancel{9}} = \frac{2}{9}$ $\quad$ Simplify

$e^{x-8} = 2/9$ $\quad$ Apply ln

$\ln(e^{x-8}) = \ln\left(\frac{2}{9}\right)$ $\quad$ $\ln(e^x) = x$

$x - 8 = \ln(2/9)$ $\quad$ Add 8

$x - 8 + 8 = \ln(2/9) + 8$ $\quad$ Simplify

$\boxed{x = \ln(2/9) + 8}$

13. Solve the equation for a.

$$4x - 16 = -2 + a.$$ Add 2

$4x-16+2 = -2+a+2$ Simplify

$4x-14=a$

$\boxed{a=4x-14}$

14. Explain how the graph of $g(x) = (x+5)^3 - 7$ is obtained from the graph of $f(x) = x^3$.

Possibilities:

(a) Shift right 5 units and shift down 7 units.

(b) Shift left 7 units and shift down 5 units.

(c) Shift right 7 units and shift up 5 units.

(d) Shift right 5 units and shift up 7 units.

$\boxed{\text{(e)}}$ Shift left 5 units and shift down 7 units.

Algebra	New Function	Geometry
Replace x with $x+5$	$f(x+5)=(x+5)^3$	Shift left 5 units
Subtract 7	$f(x+5)-7=(x+5)^3-7$	Shift down 7 units

Note: This is $g(x)$

15. Find the slope of the line through the points $(1,4)$ and $(-9,4)$.

x_1, y_1 x_2, y_2

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-4}{-9-1} = \frac{0}{-10} = 0$

16. Express the equation in logarithmic form.

$$8^3 = 512$$

Recall: $y=\log_a(x)$ means $a^y = x$

$3 = \log_8(512) \Longleftarrow 8^3 = 512$

Possibilities:

$\boxed{\text{(a) } \log_8 512 = 3}$

(b) $\log_8 3 = 512$

(c) $\log_3 512 = 8$

(d) $\log_3 8 = 512$

(e) $\log_{512} 3 = 8$

17. The endpoints of a diameter of a circle are $A(8,-9)$ and $B(-6,-2)$. Find the center of the circle.

x_1, y_1 x_2, y_2

Possibilities:

(a) $(-7, 7/2)$

$\boxed{\text{(b) } (1, -11/2)}$

(c) $(0, 7\sqrt{5})$

(d) $(7\sqrt{5}, 0)$

(e) $(7, -7/2)$

Center of the Circle = midpoint from A to B $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{8+(-6)}{2}, \frac{-9+-2}{2}\right)$

$= \left(\frac{2}{2}, \frac{-11}{2}\right) = \left(1, \frac{-11}{2}\right)$

18. Carol has \$4000. She invests x dollars at a simple interest rate of 4% and the rest of her money at a simple interest rate of 6%. After one year, the total interest earned on these investments is \$180.00. Which of the equations below would you solve to find x?

Possibilities:

(a) $\frac{x}{4} + \frac{4000-x}{6} = 180.00$

(b) $4x + 6(4000-x) = 180.00$

(c) $0.04x \times 0.06(4000-x) = 180.00$

(d) $\boxed{0.04x + 0.06(4000-x) = 180.00}$

(e) $\frac{x}{0.04} + \frac{4000-x}{0.06} = 180.00$

Let x be the amount invested at 4%

Let y be the amount invested at 6%

Total Amount invested: $x + y = 4000 \xrightarrow{\text{Solve for } y} y = 4000 - x$

Interest Amount: $\underbrace{0.04x}_{\text{Interest from the 4\% account}} + \underbrace{0.06y}_{\text{Interest from the 6\% account}} = 180$ Substitute

$0.04x + 0.06(4000-x) = 180$

19. Solve the inequality $x^2 - 5x - 15 \geq 9$. Write the solution set in interval notation.

Possibilities:

(a) $(-\infty, -8] \cup [3, \infty)$

(b) $\boxed{(-\infty, -3] \cup [8, \infty)}$

(c) $[-3, 8]$

(d) $[9, \infty)$

(e) $(-\infty, 9]$

$x^2 - 5x - 15 \geq 9$ Subtract 9 (same)

$x^2 - 5x - 15 - 9 \geq 9 - 9$ Simplify (same)

$x^2 - 5x - 24 \geq 0$ Factor

$(x-8)(x+3) \geq 0$

Critical Numbers $(x-8)(x+3) = 0$

$x - 8 = 0$ or $x + 3 = 0$

$+8 \quad +8 \qquad -3 \quad -3$

$x = 8 \qquad x = -3$

−4 0 9

−3 8

Answer: $(-\infty, -3] \cup [8, \infty)$

Test Point	$x-8$	$x+3$	Sign
−4	−	−	+
0	−	+	−
9	+	+	+

20. Which of the following statements are true?

(I) $2^{\log_2(5)} = 5$

(II) $\log(\frac{a}{b}) = \log(a) - \log(b)$ for all positive a and b.

(III) $\ln(a+b) = (\ln(a))(\ln(b))$ for all positive a and b.

Possibilities:

(a) Only (III) is true.

(b) Only (I) and (III) are true.

(c) Only (I) is true.

(d) Only (I) and (II) are true.

(e) Statements (I), (II), and (III) are all true.

See work from Review #3
Problem# 49

21. Find a formula for the parabola with vertex $(-5, 6)$ and that passes through the point $(-4, 9)$.

$y = 3(x+5)^2 + 6$ See work on Next Page

22. A ball is thrown straight upward at an initial speed of 240 feet per second. From Physics, we know that the ball will reach a height of h feet after t seconds where h and t are related by the following formula:

$$h = -16t^2 + 240t.$$

In order to determine when the ball hits the ground, you need to:

Possibilities:

(a) Find the positive h intercept of the graph of $h = -16t^2 + 240t$.

(b) Calculate the average rate of change of h with respect to t.

(c) Find the slope of a line.

$\boxed{\text{(d)}}$ Find the positive t intercept of the graph of $h = -16t^2 + 240t$. t-intercept is when h = 0. height zero means the ball is on the ground.

(e) Find the intervals where the graph of h is increasing and where it is decreasing.

23. Let $f(x) = \dfrac{1}{\sqrt{x-2}}$. Find the domain of $f(x)$.

Possibilities:

(a) $[2, \infty)$

(b) $(-2, \infty)$

(c) $(-\infty, 2]$

$\boxed{\text{(d)} \ (2, \infty)}$

(e) $(-\infty, -2)$

1st Need $\sqrt{x-2} \neq 0$ because of the fraction

So Solve: $\sqrt[2]{x-2} = 0$ — $\sqrt[n]{a} = b$ means $b^n = a$

$0^2 = x-2$ Simplify

$0 = x-2$ Add 2

$0 + 2 = x - 2 + 2$ Simplify

$2 = x$

So $x \neq 2$

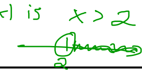
2nd Need $x - 2 \geq 0$ because of the square root. So solve

$x - 2 \geq 0$ Add 2 (same)

$x - 2 + 2 \geq 0 + 2$ Simplify

$x \geq 2$

However $x \neq 2$ so the domain of $f(x)$ is $x > 2$

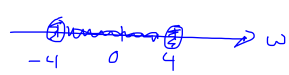


$(2, \infty)$

24. Solve the inequality and graph the solution set on the real number line.

$$|x - 2| \leq 4$$

Let $w = x - 2$ then $|w| \leq 4$



So $-4 \leq w \leq 4$ Replace w with $x-2$

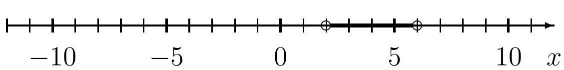
$-4 \leq x - 2 \leq 4$ Add 2 (same)

$-4 + 2 \leq x - 2 + 2 \leq 4 + 2$ Simplify (same)

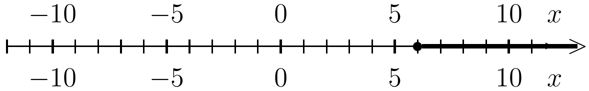
$-2 \leq x \leq 6$

Possibilities:

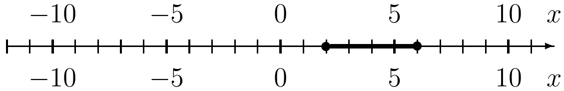
(a)



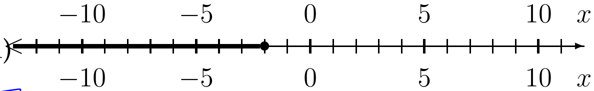
(b)



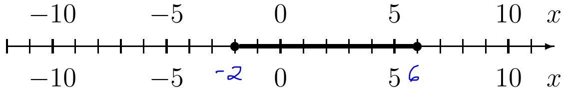
(c)



(d)



$\boxed{\text{(e)}}$



#21 Vertex = $(-5, 6)$ (h, K)

Point = $(-4, 9)$ (x, y)

Standard Form of a quadratic function: $y = a(x-h)^2 + K$

I like to call this "vertex form"

Vertex $h=-5, K=6$

$y = a(x-(-5))^2 + 6$

Simplify

* only need the value for "a"

$y = a(x+5)^2 + 6$

Point $x=-4, y=9$

$9 = a(-4+5)^2 + 6$

Simplify

$9 = a(1)^2 + 6$

$9 = a + 6$

Subtract 6

$9 - 6 = a + 6 - 6$

Simplify

$3 = a$

$a = 3$

$$\boxed{y = 3(x+5)^2 + 6}$$

25. Let $f(x) = 2x + 7$. Find $f^{-1}(5)$.

$y = f(x)$ means $f^{-1}(y) = x$

so $5 = f(x)$ means $f^{-1}(5) = x$ ← Note we are looking for "x"

However, $f(x) = 2x + 7$ so we have

$5 = f(x) = 2x + 7$

Possibilities:

(a) 6

(b) 17

(c) -2

(d) 24

$\boxed{\text{(e) -1}}$

$5 = 2x + 7$ Subtract 7

$5 - 7 = 2x + 7 - 7$ Simplify

$-2 = 2x$ Divide by 2

$\frac{-2}{2} = \frac{\cancel{2}x}{\cancel{2}}$ Simplify

$-1 = x$ Therefore $f^{-1}(5) = -1$

26. Which of the following statements are true?

(I) If $P(x) = 3x^3 + x - 15x^2 - 5$, then $P(5) = 0$. TRUE

(II) $(5, 0)$ is an x-intercept on the graph of $y = 3x^3 + x - 15x^2 - 5$. TRUE

(III) The remainder of the division problem $\dfrac{3x^3 + x - 15x^2 - 5}{x - 5}$ is zero. TRUE

(IV) $(x - 5)$ is a factor of $3x^3 + x - 15x^2 - 5$. TRUE

Possibilities:

(a) Only (III) and (VI) are true.

(b) Only (I) and (III) are true.

$\boxed{\text{(c) (I), (II), (III), and (IV) are all true.}}$

(d) Only (I) and (II) are true.

(e) None of the statements are true.

Note $P(5) = 3 \cdot 5^3 + 5 - 15 \cdot 5^2 - 5$

$= 3 \cdot 125 + 5 - 15 \cdot 25 - 5$

$= 375 + 5 - 375 - 5$

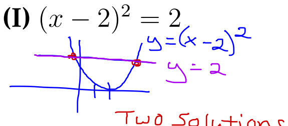
$= 380 - 375 - 5$

$= 5 - 5$

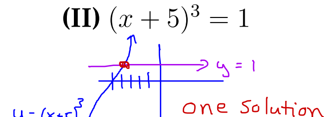
$= 0$

27. How many solutions are there for each equation?

(I) $(x - 2)^2 = 2$



(II) $(x + 5)^3 = 1$



Possibilities:

(a) Equation **(I)** has no solutions, and equation **(II)** has no solutions.

(b) Equation **(I)** has no solutions, and equation **(II)** has 1 solution.

$\boxed{\text{(c) Equation (I) has 2 solutions, and equation (II) has 1 solution.}}$

(d) Equation **(I)** has 2 solutions, and equation **(II)** has 3 solutions.

(e) Equation **(I)** has 1 solution, and equation **(II)** has 3 solutions.

28. Let $f(x) = \frac{x-3}{x-4}$. Find the x and y intercepts of the graph of $y = f(x)$.

Possibilities:

(a) x-intercept: $(3, 0)$, y-intercept: $(0, 1)$

(b) x-intercept: $(1, 0)$, y-intercept: $(0, 3)$

$\boxed{\text{(c) } x\text{-intercept: } (3, 0),\ y\text{-intercept: } (0, 3/4)}$

(d) x-intercept: $(1, 0)$, y-intercept: $(0, 1)$

(e) x-intercept: $(3/4, 0)$, y-intercept: $(0, 3)$

X-intercept: $y = 0$

$0 = \frac{x-3}{x-4}$ Multiply by $x-4$

$(x-4)\cdot 0 = \frac{x-3}{\cancel{x-4}}\cancel{(x-4)}$ Simplify

$0 = x - 3$ Add 3

$0 + 3 = x - 3 + 3$ Simplify

$3 = x$

$x = 3$ x-intercept: $(3,0)$

y-intercept: $x = 0$

$f(0) = \frac{0-3}{0-4} = \frac{-3}{-4} = \frac{3}{4}$

y-intercept: $(0, \frac{3}{4})$

29. Find the perimeter of the parallelogram ABCD with vertices A$(1, 4)$, B$(14, 4)$, C$(17, 9)$, and D$(4, 9)$.

Possibilities:

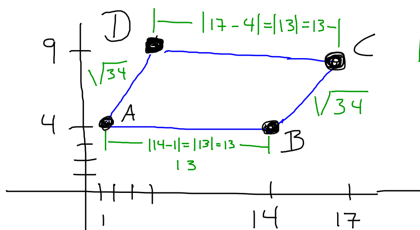
(a) 36 units

(b) 65 units

(c) $65/2$ units

(d) 238 units

$\boxed{\text{(e) } 26 + 2\sqrt{34} \text{ units}}$



$|AD| = \sqrt{(1-4)^2 + (4-9)^2} = \sqrt{(-3)^2 + (-5)^2}$
$= \sqrt{9 + 25} = \sqrt{34}$

$|BC| = \sqrt{(14-17)^2 + (4-9)^2} = \sqrt{(-3)^2 + (-5)^2}$
$= \sqrt{9 + 25} = \sqrt{34}$

Perimeter $= 13 + \sqrt{34} + 13 + \sqrt{34} = 26 + 2\sqrt{34}$ units

30. If the GGMC corporation produces x kilograms of gadgets, then their revenue, in dollars, is given by $R(x) = 100 + 800x - .5x^2$. What is the maximum revenue and how many kilograms of gadgets should be manufactures to obtain this maximum?

parabola opens downward because $a = -0.5 < 0$

1^{st} Compute: $\frac{-b}{2a} = \frac{-800}{2(-0.5)} = \frac{-800}{-1} = 800$ Where the maximum occurs

2^{nd} Compute $R(\frac{-b}{2a}) = R(800) = 100 + 800 \cdot 800 - 0.5(800)^2$
$= 100 + 640{,}000 - 320{,}000$
$= \boxed{320{,}100}$

Maximum Revenue

Maximum Revenue: 320,000

Kilograms of Gadgets: 800

31. Let $f(x) = 3x + 2$. Find the average rate of change of $f(x)$ from $x = \overset{a}{-5}$ to $x = \overset{b}{9}$.

Possibilities:

Short answer: For a line the average rate of change is the slope. Consequently, in this case we have 3.

(a) 6

(b) 2

(c) 4

(d) 5

$\boxed{\text{(e) } 3}$

long answer: 1^{st} Compute $f(9) = 3 \cdot 9 + 2 = 27 + 2 = 29$
and $f(-5) = 3 \cdot (-5) + 2 = -15 + 2 = -13$

This is what you will need to do if the function is not a line. :)

Average Rate of change $= \frac{f(b) - f(a)}{b - a} = \frac{f(9) - f(-5)}{9 - (-5)} = \frac{29 - (-13)}{9 - (-5)} = \frac{29 + 13}{9 + 5} = \frac{42}{14} = 3$ same

32. Solve.

$$\log_8(x-5)+\log_8(x+2)=1$$

Possibilities:

(a) $x_1 = 6$ and $x_2 = -3$

(b) $x_1 = 6$ and $x_2 = 3$

(c) $x_1 = 5$ and $x_2 = -2$

(d) $x_1 = 5$

(e) $x_1 = 6$

See Review# 3 problem #7 Again ☺

33. If the GGMC corporation produces x kilograms of gadgets, then their revenue, in dollars, is given by $R(x) = 100 + 800x - .3x^2$. What is the maximum revenue and how many kilograms of gadgets should be manufactures to obtain this maximum?

Maximum Revenue: 533,433.33

Kilograms of Gadgets: 1,333.33

34. Find the quotient and the remainder of the divsion problem.

Quotient = $6x^2+27x+113$
Remainder = 446

$$\frac{6x^3+3x^2+5x-6}{x-4}$$

See work on next page.

35. Find all the solutions of the system of equations.

Solutions: $(-3,-5)$ and $(6,22)$

$$\begin{cases} -3x+y=4 \\ x^2-y=14 \end{cases}$$

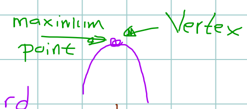
See work on next page.

36. Let $f(x) = \ln(3x+7)$. Find $f^{-1}(x)$. $f^{-1}(x) = \frac{e^x-7}{3}$ See Review #3 Problem # 12

37. Let $f(x) = 2e^{5x+3}$. Find $f^{-1}(x)$. $f^{-1}(x) = \frac{\ln\left(\frac{x}{2}\right)-3}{5}$ See Review #3 Problem #13

$\boxed{\#33}$ $R(x)=100+800x-0.3x^2$

parabola opens downward because $a=-0.3<0$



1st Compute: $\frac{-b}{2a}=\frac{-800}{2(-0.3)} = \frac{-800}{-0.6} = \frac{800}{\frac{6}{10}} = \frac{800}{1}\cdot\frac{10}{6}=\frac{8000}{6}$

$=\frac{4000}{3}\approx 1333.33$ Kilograms

Where the maximum occurs

2nd Compute $R\left(\frac{-b}{2a}\right)=R\left(\frac{4000}{3}\right)=100+800\cdot\left(\frac{4000}{3}\right)-\frac{3}{10}\left(\frac{4000}{3}\right)^2$

$=\frac{100}{1}\cdot\frac{3}{3}+\frac{3200000}{3}-\frac{\cancel{3}}{\cancel{10}}\cdot\frac{1600000\cancel{0}}{\cancel{9}\,3}$

$=\frac{300}{3}+\frac{3,200,000}{3}-\frac{1,600,000}{3}$

$=\frac{1,600,300}{3}\approx 533,433.33$

Maximum Revenue

$\boxed{\#34}$

$$\begin{array}{r|l}
 & 6x^2+27x+113 \\
x-4 & 6x^3+3x^2+5x-6 \\
 & -(6x^3-24x^2) \downarrow \quad \downarrow \\
\hline
 & 27x^2+5x-6 \\
 & -(27x^2-108x) \downarrow \\
\hline
 & 113x-6 \\
 & -(113x-452) \\
\hline
 & 446
\end{array}$$

Quotient $=6x^2+27x+113$

Remainder $=446$

$\boxed{\#35}$ Elimination Method

$$\begin{cases} -3x+y=4 \\ x^2-y=14 \end{cases}$$

$x^2-3x+0y=18$ Simplify

$x^2-3x=18$ Subtract 18

$x^2-3x-18=18-18$ Simplify

$x^2-3x-18=0$ Factor

$x^2-6x+3x-18=0$

$x(x-6)+3(x-6)=0$

$(x+3)(x-6)=0$ Zero Product Rule

$x+3=0$ or $x-6=0$

$-3 \quad -3$ $\qquad +6 \quad +6$

$x=-3$ $\qquad x=6$

Back Substitute to solve for y (I will use equation 1 but either can be used)

If $x=-3$ then $-3(-3)+y=4$

$9+y=4$

$-9 \qquad -9$

$y=-5$

So one solution is $(-3,-5)$

If $x=6$ then $-3\cdot6+y=4$

$-18+y=4$

$+18 \qquad +18$

$y=22$

So another solution is $(6,22)$

38. Which of the following functions are one-to-one?

$$f(x) = x^2 + 3 \qquad g(x) = x^3 \qquad h(x) = 3x - 9$$

Possibilities:

See Review #3 Problem # 41

(a) Only $h(x)$ is one-to-one.
(b) Only $g(x)$ and $h(x)$ are one-to-one.
(c) Only $f(x)$ and $g(x)$ are one-to-one.
(d) None of the functions are one-to-one.
(e) All of the functions are one-to-one.

---

39. Find all real solutions or state that there are NONE.

$X = \frac{-22}{5}$

$$-6x - 42 = 4x + 2.$$

See Work on Next Page

---

40. Let $P(x) = 4x^3 + 32x^2 - 236x + 312$. Decide which of the statements below are true.

Equivalent

(I) $x = 2$ is a zero of $f(x)$. TRUE
(II) $x = -2$ is a zero of $f(x)$. False
(III) $x + 2$ is a factor of $f(x)$. False
(IV) $x - 2$ is a factor of $f(x)$. TRUE
(V) The graph of $y = P(x)$ has an x-intercept at $(2, 0)$. TRUE
(VI) The graph of $y = P(x)$ has an x-intercept at $(-2, 0)$. False

Equivalent

Note $P(2) = 4 \cdot 2^3 + 32 \cdot 2^2 - 236 \cdot 2 + 312$
$= 4 \cdot 8 + 32 \cdot 4 - 472 + 312$
$= 32 + 128 - 472 + 312$
$= 160 - 472 + 312$
$= -312 + 312$
$= 0$

Possibilities:

Note $P(-2) = 4(-2)^3 + 32(-2)^2 - 236(-2) + 312$
$= 4 \cdot (-8) + 32 \cdot 4 + 472 + 312$
$= -32 + 128 + 472 + 312$
$= 96 + 472 + 312$
$= 568 + 312$
$= 880$

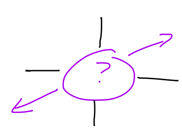
(a) (I) true; (II) false; (III) true; (IV) false; (V) true; (VI) false
(b) (I) false; (II) false; (III) false; (IV) true; (V) false; (VI) true
(c) (I) true; (II) true; (III) true; (IV) true; (V) true; (VI) true
(d) (I) true; (II) false; (III) false; (IV) true; (V) true; (VI) false
(e) (I) true; (II) false; (III) false; (IV) false; (V) false; (VI) false

---

41. Let $f(x) = 5x^{13} + 2x^4 - 6x + 1$. Determine the end behavior of $y = f(x)$.

Highest exponent is odd

Possibilities: positive leading coefficient



$y \to \infty$ as $x \to \infty$
& $y \to -\infty$ as $x \to -\infty$

(a) $y \to 1$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(b) $y \to \infty$ as $x \to \infty$ and $y \to 1$ as $x \to -\infty$
(c) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$
(d) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$
(e) $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

$\boxed{\#39}$

$-6x - 42 = 4x + 2$ Add 42

$-6x - 42 + 42 = 4x + 2 + 42$ Simplify

$-6x = 4x + 44$ Subtract $4x$

$-6x - 4x = 4x + 44 - 4x$ Simplify

$-10x = 44$ Divide by -10

$\frac{\cancel{-10}x}{\cancel{-10}} = \frac{44}{-10}$ Simplify

$\boxed{x = -\frac{22}{5}}$

Check: $-6\left(\frac{-22}{5}\right) - 42 \stackrel{?}{=} 4\left(\frac{-22}{5}\right) + 2$

$\frac{132}{5} - \frac{42}{1}\cdot\frac{5}{5} \stackrel{?}{=} \frac{-88}{5} + \frac{2}{1}\cdot\frac{5}{5}$

$\frac{132}{5} - \frac{210}{5} \stackrel{?}{=} \frac{-88}{5} + \frac{10}{5}$

$-\frac{78}{5} = \frac{-78}{5}$ ☺ Consequently, $x = \frac{-22}{5}$ is a solution.

42. Find the area of the triangle with vertices $A(-2, 3)$, $B(11, 3)$, and $C(3, 8)$.

Possibilities:

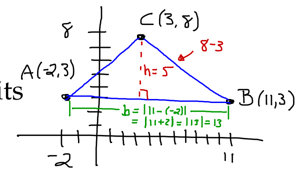
(a) 65 square units

(b) $13 + \sqrt{89} + 5\sqrt{2}$ square units

(c) 66 square units

(d) 72 square units

(e) 65/2 square units



Area of a triangle $= \frac{1}{2} b h$

$= \frac{1}{2} \cdot 13 \cdot 5$

$= \boxed{\frac{65}{2} \text{ Square units}}$

43. The number of bacteria in a culture is modeled by the function $n(t) = 50e^{.45t}$ where t is measured in hours. After how many hours ~~will~~ until the number of bacteria reach 3000?

Possibilities:

(a) About 3.95 hours

(b) About 10.02 hours

(c) About 9.10 hours

(d) About 49.05 hours

(e) About 4.09 hours

$3000 = 50e^{0.45t}$ Divide by 50

$\frac{3000}{50} = \frac{\cancel{50}e^{0.45t}}{\cancel{50}}$ Simplify

$60 = e^{0.45t}$ Apply ln

$\ln(60) = \ln(e^{0.45t})$ $\ln(e^x) = x$

$\ln(60) = 0.45t$ Divide by 0.45

$\frac{\ln(60)}{0.45} = \frac{\cancel{0.45}t}{\cancel{0.45}}$ Simplify

$t = \frac{\ln(60)}{0.45}$

$t \approx 9.10$ hours

44. Find the remainder of the divsion problem.

$$\frac{x^4 - 21x^3 + 153x^2 - 455x + 450}{x - 9}$$

The remainder is zero and $P(9) = 0$

See work on Next Page

Let $P(x) = x^4 - 21x^3 + 153x^2 - 455x + 450$. What is $P(9)$? What is the relationship between $P(9)$ and the remainder? They are the same

45. If \$3000 is invested at an interest rate of 8% per year compounded quarterly, find the amount of the investment at the end of 13 years.

$P_0 = 3000$ $r = 0.08$ $n = 4$ $t = 13$

Possibilities:

(a) \$3247.30

(b) \$4081.47

(c) \$3880.82

(d) \$8400.98

(e) \$164118.12

$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt}$

$P(13) = 3000\left(1 + \frac{0.08}{4}\right)^{4 \cdot 13}$

$= 3000(1.02)^{52}$

$= \boxed{8400.98}$

$\boxed{\#44}$

$$\begin{array}{r} x^3 - 12x^2 + 45x - 50 \\ x-9 \overline{\big) x^4 - 21x^3 + 153x^2 - 455x + 450} \\ \underline{-x^4 + 9x^3 \qquad\qquad\qquad\qquad} \\ -12x^3 + 153x^2 - 455x + 450 \\ \underline{+12x^3 - 108x^2 \qquad\qquad\quad} \\ 45x^2 - 455x + 450 \\ \underline{-45x^2 + 405x \qquad} \\ -50x + 450 \\ \underline{+50x - 450} \\ 0 \end{array}$$

$$\begin{aligned} P(9) &= 9^4 - 21(9)^3 + 153(9)^2 - 455(9) + 450 \\ &= 6561 - 21(729) + 153(81) - 4095 + 450 \\ &= 6561 - 15309 + 12393 - 4095 + 450 \\ &= -8748 + 12393 - 4095 + 450 \\ &= 3645 - 4095 + 450 \\ &= -450 + 450 \\ &= 0 \end{aligned}$$

Same as the remainder

46. How many solutions does the following system of equation have?

Elimination

$$\begin{cases} 3x + 8y = 16 \xrightarrow{\text{multiply by two}} \\ 6x - 16y = 32 \longrightarrow \end{cases}$$

$6x + 16y = 32$
$6x - 16y = 32$
$12x + 0y = 64$ Simplify
$12x = 64$ Divide by 12
$\frac{\cancel{12}x}{\cancel{12}} = \frac{64}{12}$ Simplify
$x = \frac{16}{3}$

Possibilities:

(a) No solutions

$\boxed{\text{(b) One solution}}$

(c) Two solutions

(d) Three solutions

(e) Infinitely many solutions

$\cancel{3}\left(\frac{16}{\cancel{3}}\right) + 8y = 16$
$16 + 8y = 16$
$16 + 8y - 16 = 16 - 16$
$8y = 0$
$\frac{8y}{8} = \frac{0}{8}$
$y = 0$

$\left(\frac{16}{3}, 0\right) \leftarrow$ one solution

47. Let $f(x) = 3x^2 + 5x$. Find $\dfrac{f(x+h) - f(x)}{h}$.

Possibilities: 1st Compute $f(x+h) = 3(x+h)^2 + 5(x+h) = 3(x+h)(x+h) + 5x + 5h$
$= 3(x^2 + xh + hx + h^2) + 5x + 5h = 3(x^2 + 2xh + h^2) + 5x + 5h$
$= 3x^2 + 6xh + 3h^2 + 5x + 5h$

(a) $\dfrac{3h^2 + 5h}{h}$

(b) $\dfrac{6xh + 3h^2 + 10x + 5h}{h}$

$\boxed{\text{(c) } 6x + 3h + 5}$

(d) $-6x - 3h - 5$

(e) $3h^2 + 5h$

$\frac{f(x+h) - f(x)}{h} = \frac{(3x^2 + 6xh + 3h^2 + 5x + 5h) - (3x^2 + 5x)}{h}$
$= \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{5x} + 5h - \cancel{3x^2} - \cancel{5x}}{h} = \frac{6xh + 3h^2 + 5h}{h}$
$= \frac{\cancel{h}(6x + 3h + 5)}{\cancel{h}} = 6x + 3h + 5$

48. Suppose that the graph of $y = f(x)$ contains the point (5,3). Find a point on the graph of $y = 2f(x) - 4$.

(5, 2) See Review #3 Problem #2

49. Let $r(x) = \frac{x+1}{x^2-4}$. Find the horizontal asymptotes of $r(x)$.

Possibilities: Note bottom degree is larger than top degree

(a) $r(x)$ does not have any horizontal asymptotes.

(b) $y = 1$

$\boxed{\text{(c) } y = 0}$

(d) $x = 2$ and $x = -2$

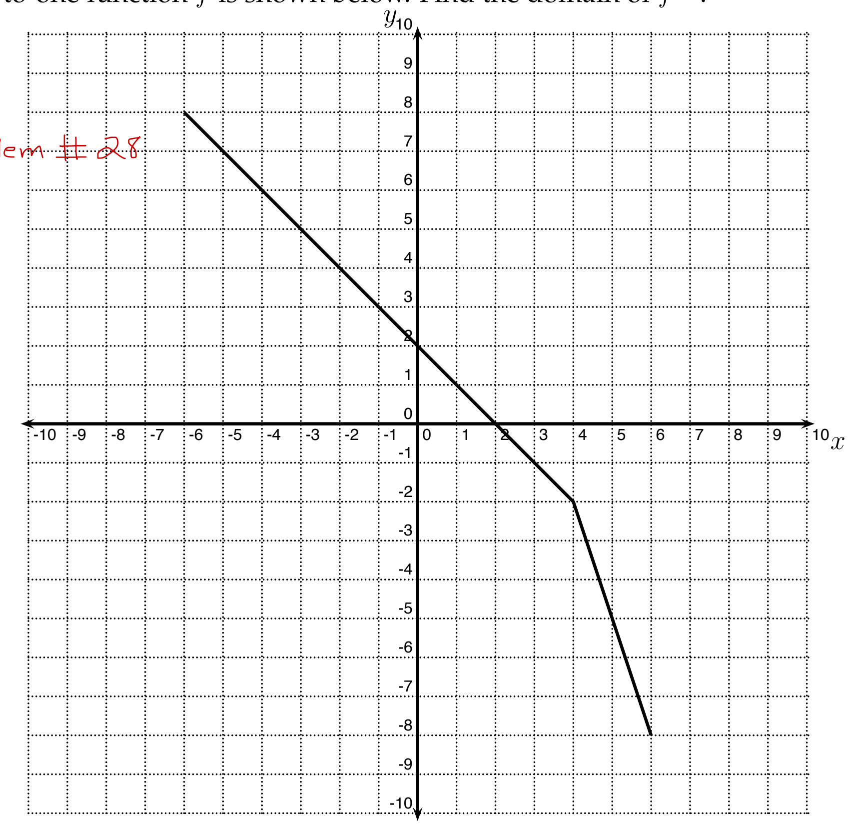
(e) $x = 1$

So $y = 0$ is the horizontal asymptote of r

50. The graph of the one-to-one function f is shown below. Find the domain of f^{-1}.

[-8, 8]

See Review #3 Problem #28



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51. Find the remainder of the divsion problem.

See work on next page

Remainder = 2496

$$\frac{x^4 - 16x^3 + 71x^2 - 56x - 144}{x+4}$$

Let $P(x) = x^4 - 16x^3 + 71x^2 - 56x - 144$. What is $P(-4)$? What is the relationship between $P(-4)$ and the remainder?

$P(-4) = 2496$

They are the same

---

52. $\log\left(\frac{x^{-3}}{y^5z^8}\right) = \log(x^{-3}) - \log(y^5 \cdot z^8)$

$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$

$\log_a(m \cdot n) = \log_a(m) + \log_a(n)$

$= \log(x^{-3}) - [\log(y^5) + \log(z^8)]$

Distribute

$= \log(x^{-3}) - \log(y^5) - \log(z^8)$

$= -3\log(x) - 5\log(y) - 8\log(z)$

$\log_a(m^r) = r\log_a(m)$

Possibilities:

(a) $-3\log(x) + 5\log(y) - 8\log(z)$

(b) $-3\log(x)/5\log(y) * 8\log(z)$

(c) $-3\log(x) - 5\log(y) + 8\log(z)$

(d) $\boxed{-3\log(x) - 5\log(y) - 8\log(z)}$

(e) $-3\log(x)/(5\log(y) * 8\log(z))$

$\boxed{\#51}$

$$
\begin{array}{r|l}
 & x^3 - 20x^2 + 151x - 660 \\
\hline
x+4 & x^4 - 16x^3 + 71x^2 - 56x - 144 \\
 & -x^4 - 4x^3 \\
\hline
 & -20x^3 + 71x^2 - 56x - 144 \\
 & +20x^3 + 80x^2 \\
\hline
 & 151x^2 - 56x - 144 \\
 & -151x^2 - 604x \\
\hline
 & -660x - 144 \\
 & +660x + 2640 \\
\hline
 & 2496
\end{array}
$$

Remainder $= 2496$

Note $P(-4) = (-4)^4 - 16(-4)^3 + 71(-4)^2 - 56(-4) - 144$

$= 256 + 16 \cdot 64 + 71 \cdot 16 + 224 - 144$

$= 256 + 1024 + 1136 + 224 - 144$

$= 1280 + 1136 + 224 - 144$

$= 2416 + 224 - 144$

$= 2640 - 144$

$= 2496$

Same as Remainder

53. Let $f(x) = x^2 - 8x + 7$. Find the y-intercept(s) of the graph of $f(x)$.

Possibilities:

(a) Both $(7, 0)$ and $(1, 0)$

$\boxed{\text{(b) Only } (0, 7)}$

(c) Only $(7, 0)$

(d) Both $(0, 7)$ and $(0, 1)$

(e) Both $(-7, 0)$ and $(-1, 0)$

y-intercept : $x=0$

$f(0) = 0^2 - 8(0) + 7 = 0 - 0 + 7 = 0 + 7 = 7$

So $(0,7)$ is the only y-intercept

54. A ball is thrown straight upward at an initial speed of 200ft/sec. From Physics it is known that, after t seconds, the ball reaches a height h feet given by the formula

$$h = -16t^2 + 200t.$$

this is a parabola so the maximum height will be the y coordinate of the vertex

What is the maximum height reached by the ball?

Possibilities:

(a) 7.55 ft

(b) 12.50 ft

(c) 6.25 ft

$\boxed{\text{(d) } 625.00 \text{ ft}}$

(e) 677.30 ft

1st Compute $\frac{-b}{2a} = \frac{-200}{2(-16)} = \frac{200}{32} = \frac{25}{4}$

So the maximum height occurs at $t = \frac{25}{4}$ seconds and
and the maximum height is $-16\left(\frac{25}{4}\right)^2 + 200\left(\frac{25}{4}\right)$

$= -\frac{\cancel{16}}{1} \cdot \frac{625}{\cancel{16}} + \frac{5000}{4}$

$= -625 + 1250$

$= 625$ feet

55. Find an equation for the line through the points $\overset{x_1\ y_1}{(-2, 5)}$ and $\overset{x_2\ y_2}{(5, 14)}$.

Possibilities:

(a) $y + 5 = \frac{9}{7}(x - 2)$

(b) $y - 5 = \frac{9}{7}(x - 14)$

$\boxed{\text{(c) } y - 5 = \frac{9}{7}(x + 2)}$

(d) $y - 14 = -\frac{7}{9}(x - 5)$

(e) $y - 2 = -\frac{7}{9}(x - 5)$

1st Compute the slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 5}{5 - (-2)} = \frac{14-5}{5+2} = \frac{9}{7}$

Use Point-slope form : $y - y_1 = m(x - x_1)$

use either point:

If one uses $(-2,5)$ then $y - 5 = \frac{9}{7}(x - (-2))$

$y - 5 = \frac{9}{7}(x + 5)$

56. Let $f(x) = \log_2(x + 7) - 3$. Find $f^{-1}(x) = 2^{x+3} - 7$ See work on next page

57. You wish to purchase a new cell phone. You have a coupon for \$10 and the store is running a special which allows you to deduct 15% from the price. If the original price of the cell phone is 175 dollars, what is the final price if you apply the coupon and then apply the 15% discount.

$\boxed{\$140.25}$ See work from Review #3 Problem #19

#56 $f(x) = \log_2(x+7) - 3$

$y = \log_2(x+7) - 3$

$x = \log_2(y+7) - 3$

1st Replace $f(x)$ with y

2nd Interchange x and y

3rd Solve for y

$x = \log_2(y+7) - 3$ — Add 3

$x + 3 = \log_2(y+7) - 3 + 3$ — Simplify

$x + 3 = \log_2(y+7)$ — $y = \log_a(x)$ means $a^y = x$

$2^{x+3} = y + 7$ — Subtract 7

$2^{x+3} - 7 = y + 7 - 7$ — Simplify

$2^{x+3} - 7 = y$ — 4th Replace y with $f^{-1}(x)$

$$\boxed{f^{-1}(x) = 2^{x+3} - 7}$$

58. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple? Round your answer to the nearest tenth of a year.

$P_0 = 5000$ $\quad r = 0.05$

Possibilities:

(a) 20.0 years

(b) 32.2 years

(c) 22.0 years

(d) 13.9 years

(e) 27.7 years

$P(t) = 4 \cdot 5000 = 20{,}000$ $\qquad P(t) = P_0 e^{rt}$

$P(t) = P_0 e^{rt}$

$20000 = 5000 e^{0.05t}$ Divide by 5000

$\frac{20000}{5000} = \frac{\cancel{5000} e^{0.05t}}{\cancel{5000}}$ Simplify

$4 = e^{0.05t}$ Apply ln

$\ln(4) = \ln(e^{0.05t})$ $\quad \ln(e^x) = x$

$\ln(4) = 0.05t$ Divide by 0.05

$\frac{\ln(4)}{0.05} = \frac{\cancel{0.05}t}{\cancel{0.05}}$ Simplify

$t = \frac{\ln(4)}{0.05}$

$t \approx 27.7$ years

---

59. Let $r(x) = \frac{x^2+x-90}{x^2-9x+18}$. Find the vertical asymptotes of $r(x)$.

Possibilities:

(a) $x = 9$ and $x = -10$

(b) $y = 9$ and $y = -10$

(c) $x = 3$ and $x = 6$

(d) $y = 3$ and $y = 6$

(e) $r(x)$ does not have any vertical asymptotes.

Need zero's of the denominator that are not zero's of the Numerator

Note: $x^2 - 9x + 18 = (x-6)(x-3)$ so $x = 6$ & $x = 3$ are zero's of the denominator

Now Note: $x^2 + x - 90 = (x+10)(x-9)$ so $x = -10$ & $x = 9$ are the only zero's of the Numerator

---

60. Let $P(x) = 8x^7 + 4x + 7$. List all possible rational zeros of $P(x)$ given by the Rational Zeros Theorem (but do not check to see which are actually zeros).

Possibilities:

$\frac{\text{Divisors of } 7}{\text{Divisors of } 8} = \frac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 4, \pm 8} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm 7, \pm \frac{7}{2}, \pm \frac{7}{4}, \pm \frac{7}{8}$

(a) $\pm1, \pm8, \pm8/7$

(b) $\pm1, \pm2, \pm4, \pm8, \pm7, \pm7/2, \pm7/4, \pm7/8$

(c) $\pm1, \pm1/2, \pm1/4, \pm1/8, \pm7, \pm7/2, \pm7/4, \pm7/8$

(d) $\pm1, \pm2, \pm4, \pm8, \pm1/7, \pm2/7, \pm4/7, \pm8/7$

(e) $\pm1, \pm8, \pm7/8$

---

61. Let $f(x) = 2x^2 + 4x$. Find $f(x+4)$.

Possibilities:

(a) $2x^2 + 4x + 4$

(b) $2x^2 + 5x + 4$

(c) $2x^2 + 4x + 48$

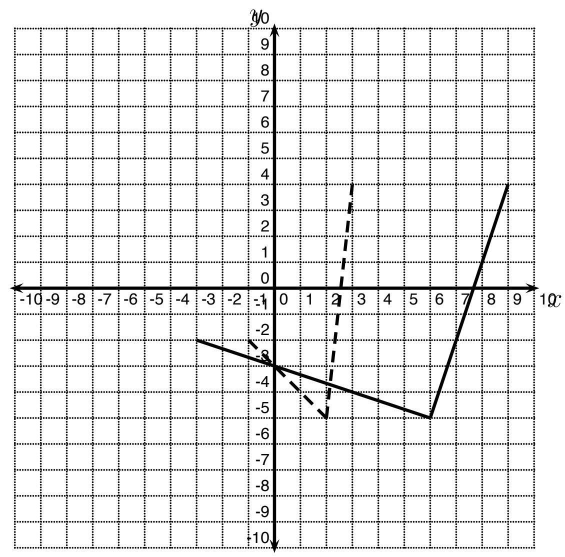
(d) $96x^2 + 192x$

(e) $2x^2 + 20x + 48$

$$\begin{aligned} f(x+4) &= 2(x+4)^2 + 4(x+4) \\ &= 2(x+4)(x+4) + 4x + 16 \\ &= 2(x^2 + 4x + 4x + 16) + 4x + 16 \\ &= 2(x^2 + 8x + 16) + 4x + 16 \\ &= 2x^2 + 16x + 32 + 4x + 16 \\ &= 2x^2 + 20x + 48 \end{aligned}$$

62. Let $f(x) = \dfrac{2x+3}{4-5x}$. Find $f^{-1}(x)$ $= \dfrac{3-4x}{-5x-2}$ See Work on next page

63. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$.



Possibilities:

(a) $g(x) = -3f(x)$

(b) $g(x) = f\left(\frac{1}{3}x\right)$

(c) $g(x) = 3f(x)$

(d) $g(x) = \frac{1}{3}f(x)$

(e) $\boxed{g(x) = f(3x)}$

See Review #3 Problem# 50

64. Find the vertex of $y = -4x^2 - 24x - 29$. $(-3, 7)$ See work on next page

65. Does the graph of $y = 3x^2 + 12x - 5$ have a maximum or a minimum? What is it?

Parabola open up-ward $\cup$ minimum because $a = 3 < 0$

66. Let $P(x) = 4x^2 + 3x + 25$. Find the y-intercept of the graph of $y = P(x)$.

y-intercept: $x = 0$

$P(0) = 4 \cdot 0^2 + 3 \cdot 0 + 25$

$= 0 + 0 + 25$

$= \boxed{25}$

$\boxed{\#62}$ $f(x) = \dfrac{2x+3}{4-5x}$ 1st Replace $f(x)$ with y

$y = \dfrac{2x+3}{4-5x}$ 2nd Interchange x and y

$x = \dfrac{2y+3}{4-5y}$ 3rd Solve for y

$x = \dfrac{2y+3}{4-5y}$ Multiply by $4-5y$

$(4-5y)\,x = \dfrac{2y+3}{\cancel{4-5y}}\,\cancel{(4-5y)}$ Simplify

$4x - 5xy = 2y+3$ Subtract $4x$

$4x - 5xy - 4x = 2y+3-4x$ Simplify

$-5xy = 2y+3-4x$ Subtract $2y$

$-5xy - 2y = 2y+3-4x-2y$ Simplify

$-5xy - 2y = 3 - 4x$ Factor

$(-5x-2)\,y = 3-4x$ Divide by $-5x-2$

$\dfrac{\cancel{(-5x-2)}\,y}{\cancel{-5x-2}} = \dfrac{3-4x}{-5x-2}$ Simplify

$y = \dfrac{3-4x}{-5x-2}$ 4th Replace y with $f^{-1}(x)$

$f^{-1}(x) = \dfrac{3-4x}{-5x-2}$

$\boxed{\#64}$ $y = -4x^2 - 24x - 29$

1st Compute $h = \dfrac{-b}{2a} = \dfrac{-(-24)}{2(-4)} = -\dfrac{24}{8} = -3$

2nd Compute $k = -4(-3)^2 - 24(-3) - 29 = -4(9) + 72 - 29$
$= -36 + 72 - 29 = 36 - 29 = 7$

So the vertex $= (-3, 7)$

67.

$$f(x) = \begin{cases} x+9 & \text{if } x \le 4 \\ x-9 & \text{if } x > 4 \end{cases}$$

Find $f(12)$.

Possibilities:

Step 1: Determine which expression to use

Is $12 \le 4$? No !!! so don't use $x+9$

Is $12 > 4$? Yes ☺ so use $x-9$

Therefore, $f(12) = 12-9 = 3$

(a) 7
(b) 63
(c) 21
(d) Both 21 and 3.
(e) $\boxed{3}$

68. Find all of the zeros of $P(x) = x^3 + 5x^2 + 4x$.

Possibilities:

Need to find when $P(x) = 0$

$x^3+5x^2+4x=0$ Factor

$x(x^2+5x+4)=0$

$x(x+4)(x+1)=0$ Zero Product Rule

$x=0$ $x+4=0$ or $x+1=0$

$-4 \; -4$ $-1 \; -1$

$x=-4$ $x=-1$

(a) 0,-1,4
(b) $\boxed{0,-1,-4}$
(c) -1,-4,1
(d) -1,-4,3
(e) 0,1,4

69. Simplify.

$$(9x+8)(2x-5) - 16x - 64$$ Distribute

$18x^2 - 45x + 16x - 40 - 16x - 64$ Combine Like terms

$18x^2 - 45x - 104$

Possibilities:

(a) $\boxed{18x^2 - 45x - 104}$
(b) $18x^2 - 29x - 32$
(c) $18x - 48$
(d) $25x + 3$
(e) $18x^2 - 45x + 24$

70. Find all real solutions or state that there are NONE.

$x=2$ and $x=5$

$$x^2 - 7x + 13 = 3.$$

See work on Next Page

71. Find all real solutions or state that there are NONE.

$x = \frac{-29}{5}$ is the only real solution

$$\frac{3}{x+6} + \frac{7}{x+7} = \frac{5}{x^2+13x+42}.$$

See work on Next Page

$\boxed{\#70}$

$x^2 - 7x + 13 = 3$ — Subtract 3

$x^2 - 7x + 13 - 3 = 3 - 3$ — Simplify

$x^2 - 7x + 10 = 0$ — Factor

$(x-5)(x-2) = 0$ — Zero Product Rule

$x - 5 = 0$ or $x - 2 = 0$

$\quad +5 \;\; +5 \qquad\qquad +2 \;\; +2$

$x = 5 \qquad\qquad x = 2$

Check $x = 5$

$5^2 - 7(5) + 13 \stackrel{?}{=} 3$

$25 - 35 + 13 \stackrel{?}{=} 3$

$-10 + 13 \stackrel{?}{=} 3$

$3 = 3$ ☺

So $x = 5$ is a solution

Check $x = 2$

$2^2 - 7(2) + 13 \stackrel{?}{=} 3$

$4 - 14 + 13 \stackrel{?}{=} 3$

$-10 + 13 \stackrel{?}{=} 3$

$3 = 3$ ☺

$\boxed{\#71}$

$$\frac{3}{x+6} + \frac{7}{x+7} = \frac{5}{x^2 + 13x + 42}$$

multiply both sides by the LCD

1st Compute the LCD

The Denominators are:

$x + 6$

$x + 7$

$\cancel{(x+6)}\cancel{(x+7)}\; x^2 + 13x + 42 = (x+6)(x+7)$

$LCD = (x+6)^{\boxed{1}}(x+7)^{\boxed{1}}$

Distribute on Left

Cancel on Right

$$(x+6)(x+7)\left[\frac{3}{x+6} + \frac{7}{x+7}\right] = \left[\frac{5}{\cancel{(x+6)(x+7)}}\right]\cancel{(x+6)(x+7)}$$

$$\cancel{(x+6)}(x+7)\frac{3}{\cancel{x+6}} + (x+6)\cancel{(x+7)}\frac{7}{\cancel{x+7}} = 5 \qquad \text{Simplify}$$

$(x+7)\cdot 3 + (x+6)\cdot 7 = 5$ — Distribute

$3x + 21 + 7x + 42 = 5$ — Simplify

$10x + 63 = 5$ — Subtract 63

$10x + 63 - 63 = 5 - 63$ — Simplify

$10x = -58$ — Divide by 10

$\dfrac{\cancel{10}x}{\cancel{10}} = \dfrac{-58}{10}$ — Simplify

$x = \dfrac{-29}{5}$

Check.

$$\frac{3}{\frac{-29}{5} + 6\cdot\frac{5}{5}} + \frac{7}{\frac{-29}{5} + 7\cdot\frac{5}{5}} \stackrel{?}{=} \frac{5}{\left(\frac{-29}{5}\right)^2 + 13\left(\frac{-29}{5}\right) + 42}$$

$$\frac{3/1}{\frac{-29+30}{5}} + \frac{7/1}{\frac{-29+35}{5}} \stackrel{?}{=} \frac{5}{\frac{841}{25} - \frac{377}{5}\cdot\frac{5}{5} + \frac{42}{1}\cdot\frac{25}{25}}$$

$$\frac{\frac{3}{1}}{\frac{1}{5}} + \frac{\frac{7}{1}}{\frac{6}{5}} \stackrel{?}{=} \frac{5}{\frac{841 - 1885 + 1050}{25}}$$

$$\frac{3}{1}\cdot\frac{5}{1} + \frac{7}{1}\cdot\frac{5}{6} \stackrel{?}{=} \frac{\frac{5}{1}}{\frac{3776}{25}}$$

$$\frac{15}{1}\cdot\frac{6}{6} + \frac{35}{6} \stackrel{?}{=} \frac{5}{1}\cdot\frac{25}{6}$$

$$\frac{90}{6} + \frac{35}{6} \stackrel{?}{=} \frac{125}{6}$$

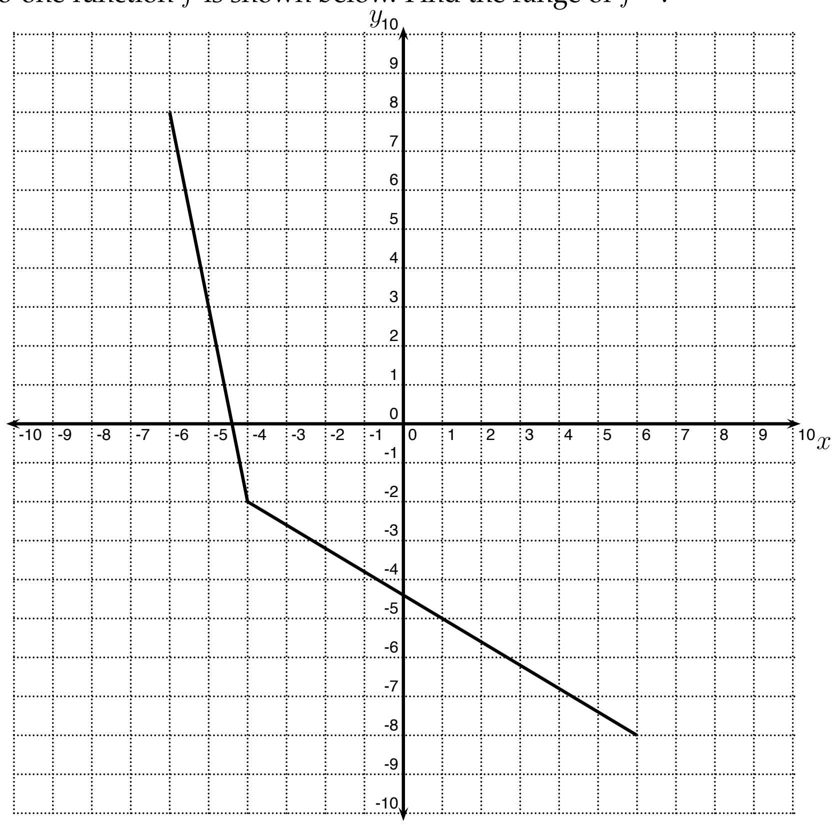
$$\frac{125}{6} = \frac{125}{6}$$ ☺

So $x = \frac{-29}{5}$ is the only solution

72. The graph of the one-to-one function f is shown below. Find the range of f^{-1}.

$[-6, 6]$

See Review #3 Problem #30



73. Solve.

$$\frac{9}{x} + \frac{4}{x-5} = 0$$

Possibilities:

(a) 9

(b) 45/13

(c) 5/13

(d) 20/13

(e) 1

See work on next Page

74. Find a polynomial of degree 3 that has zeros -3, 3, and 6 and in which the coefficient of x^2 is -60.

Polynomial: $10(x+3)(x-3)(x-6)$

or $10x^3 - 60x^2 - 90x + 540$

See work on next Page

$\boxed{\#73}$ $\dfrac{9}{x} + \dfrac{4}{x-5} = 0$

1st Compute the LCD

the denominators are:

Multiply both sides by LCD

x

$x-5$

so LCD $= x^{\boxed{1}}(x-5)^{\boxed{1}}$

$x(x-5)\left[\dfrac{9}{x} + \dfrac{4}{x-5}\right] = 0\, x(x-5)$

Distribute

$\cancel{x}(x-5)\dfrac{9}{\cancel{x}} + x\cancel{(x-5)}\dfrac{4}{\cancel{x-5}} = 0$ Simplify

$(x-5)9 + x4 = 0$

$9x - 45 + 4x = 0$

$13x - 45 = 0$ Add 45

$13x - 45 + 45 = 0 + 45$ Simplify

$13x = 45$ Divide by 13

$\dfrac{\cancel{13}x}{\cancel{13}} = \dfrac{45}{13}$ Simplify

$x = \dfrac{45}{13}$ Check

Check: $x = \dfrac{45}{13}$

$\dfrac{9}{\frac{45}{13}} + \dfrac{4}{\frac{45}{13} - \frac{5}{1}\cdot\frac{13}{13}} \stackrel{?}{=} 0$

$\dfrac{\cancel{9}}{1}\cdot\dfrac{13}{\cancel{45}_5} + \dfrac{4}{\frac{45-65}{13}} \stackrel{?}{=} 0$

$\dfrac{13}{5} + \dfrac{4}{\frac{-20}{13}} \stackrel{?}{=} 0$

$\dfrac{13}{5} - \dfrac{\cancel{4}}{1}\cdot\dfrac{13}{\cancel{20}_5} \stackrel{?}{=} 0$

$\dfrac{13}{5} - \dfrac{13}{5} \stackrel{?}{=} 0$

$0 = 0$ ☺

Consequently, $x = \frac{45}{13}$ is a solution

$\boxed{\#74}$

$y = a(x+3)(x-3)(x-6)$

$= a(x^2 - 3x + 3x - 9)(x-6)$

$= a(x^2 - 9)(x-6)$

$= a(x^3 - 6x^2 - 9x + 54)$

$= ax^3 - 6ax^2 - 9ax + 54a$

want $-6a = -60$ Divide by -6

$\dfrac{\cancel{-6}a}{\cancel{-6}} = \dfrac{-60}{-6}$ Simplify

$a = 10$

So $y = 10x^3 - 60x^2 - 90x + 540$

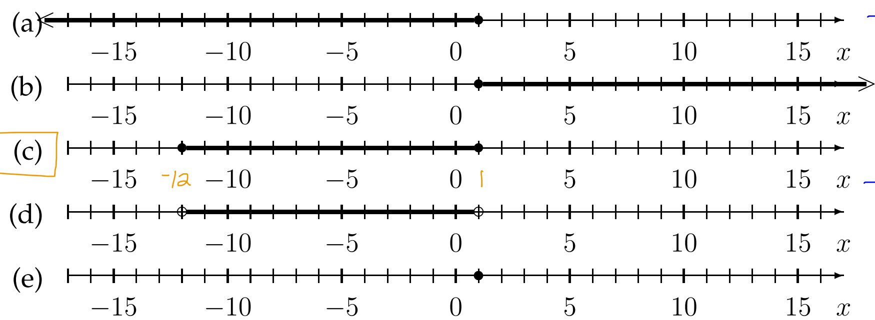
or $y = 10(x+3)(x-3)(x-6)$

75. Solve the inequality and graph the solution set on the real number line.

$$|2x + 11| \le 13$$

Let $w = 2x+11$ then $|w| \le 13$

Possibilities:



$-13 \le w \le 13$ Replace "w" with $2x+11$

$-13 \le 2x+11 \le 13$ Subtract 11 (same)

$-13-11 \le 2x+11-11 \le 13-11$ Simplify (same)

$-24 \le 2x \le 2$ Divide by 2 (same)

$\frac{-24}{2} \le \frac{2x}{2} \le \frac{2}{2}$ Simplify (same)

$-12 \le x \le 1$

---

76. Find the vertex of the parabola given by $y = 3x^2 - 5x + 9$.

Possibilities:

1st Compute $h = \frac{-b}{2a} = \frac{-(-5)}{2(3)} = \frac{5}{6}$

(a) $(-5/6, -83/12)$

$\boxed{\text{(b) } (5/6, 83/12)}$

2nd Compute $k = f\left(\frac{-b}{2a}\right) = f\left(\frac{5}{6}\right) = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 9$

(c) $(61/4, 5/6)$

(d) $(83/12, 5/6)$

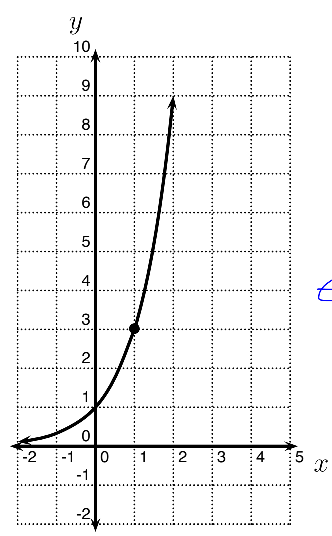
(e) $(-5/6, 61/4)$

$= \frac{\cancel{3}}{1} \cdot \frac{25}{\cancel{36}_{12}} - \frac{25}{6} + \frac{9}{1} = \frac{25}{12} - \frac{25}{6} \cdot \frac{2}{2} + \frac{9}{1} \cdot \frac{12}{12}$

$= \frac{25}{12} - \frac{50}{12} + \frac{108}{12} = \frac{-25}{12} + \frac{108}{12} = \boxed{\frac{83}{12}}$

---

77. The graph of an exponential function, $f(x) = a^x$ is shown below. Find a.



Note $f(1) = 3$ by the graph below

However, $f(1) = a^1 = a$

So $a = f(1) = 3$

Therefore $a = 3$

Possibilities:

(a) $\frac{1}{4}$

$\boxed{\text{(b) } 3}$

(c) $\frac{1}{3}$

(d) 4

(e) 2

78. Let $P(x) = x^4 - 17x^2 - x^3 - 3x - 60$. Find the real zeros of P(x). $X = -4$ & $x = 5$ See work on Next page

79. Let $P(x) = 2x^3 + x^2 - 16x - 15$. Find the real zeros of P(x). $X = -1$, $x = 3$ & $x = \frac{-5}{2}$ See work on following Page

80. Does the graph of $P(x) = (x-5)^{900}(x+4)^{901}$ cross the x-axis at $x = 5$? $X=5$ is a root of multiplicity 900 which is even so the graph touches the x-axis @ $x=5$

81. Does the graph of $P(x) = (x-5)^{900}(x+4)^{901}$ cross the x-axis at $x = -4$? $X=-4$ is a root of multiplicity 901 which is odd so the graph crosses the x-axis @ $x=-4$

82. Find the remainder of the divsion problem.

Remainder is zero
$P(9) = 0$

$$\frac{x^4 - 24x^3 + 207x^2 - 756x + 972}{x - 9}$$

See work on following Page

Let $P(x) = x^4 - 24x^3 + 207x^2 - 756x + 972$. What is $P(9)$? What is the relationship between $P(9)$ and the remainder? They are the Same

83. Find the quotient and the remainder of the divsion problem.

Quotient $= x^3 - 9x^2 + 24x - 16$
Remainder $= 0$

$$\frac{x^4 - 18x^3 + 105x^2 - 232x + 144}{x - 9}$$

See work on following Page

84. Determine the end behavior of P(x)=$3x^{56} - 7x^3 + 21$.

highest exponent is even
positive leading coefficient

$y \to \infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

85. Determine the end behavior of P(x)=$5x - 3 - x^{99}$.

highest exponent is odd
Negative leading coefficient

$y \to -\infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

$\boxed{\#78}$ $P(x) = x^4 - 17x^2 - x^3 - 3x - 60$

-4 works

5 works too

By the rational Root Theorem the possible rational roots are $= \dfrac{\text{divisors of } 60}{\text{divisors of } 1} = \dfrac{\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm10, \pm12, \pm15, \pm20, \pm30, \pm60}{\pm1}$

$P(1) = 1^4 - 17(1)^2 - 1^3 - 3(1) - 60 = -80$

$P(-1) = (-1)^4 - 17(-1)^2 - (-1)^3 - 3(-1) - 60 = -72$

$P(2) = 2^4 - 17(2) - 2^3 - 3(2) - 60 = -126$

$P(-2) = (-2)^4 - 17(-2) - (-2)^3 - 3(-2) - 60 = -98$

$P(3) = 3^4 - 17(3)^2 - 3^3 - 3\cdot3 - 60 = -168$

$P(-3) = (-3)^4 - 17(-3)^2 - (-3)^3 - 3(-3) - 60 = -96$

$P(4) = 4^4 - 17(4)^2 - 4^3 - 3(4) - 60 = -152$

$P(-4) = (-4)^4 - 17(-4)^2 - (-4)^3 - 3(-4) - 60 = 0$ ☺

$P(5) = 5^4 - 17(5)^2 - 5^3 - 3(5) - 60 = 0$ ☺

$P(-5) = (-5)^4 - 17(-5)^2 - (-5)^3 - 3(-5) - 60 = 280$

$P(6) = 6^4 - 17(6)^2 - 6^3 - 3(6) - 60 = 390$

$P(-6) = (-6)^4 - 17(-6)^2 - (-6)^3 - 3(-6) - 60 = 858$

$P(10) = 10^4 - 17(10)^2 - 10^3 - 3(10) - 60 = 7210$

$P(-10) = (-10)^4 - 17(-10)^2 - (-10)^3 - 3(-10) - 60 = 9270$

$P(12) = (12)^4 - 17(12)^2 - 12^3 - 3(12) - 60 = 16464$

$P(-12) = (-12)^4 - 17(-12)^2 - (-12)^3 - 3(-12) - 60 = 19992$

$P(15) = 15^4 - 17(15)^2 - 15^3 - 3(15) - 60 = 43320$

$P(-15) = (-15)^4 - 17(-15)^2 - (-15)^3 - 3(-15) - 60 = 50160$

$P(20) = 20^4 - 17(20)^2 - 20^3 - 3(20) - 60 = 145080$

$P(-20) = (-20)^4 - 17(-20)^2 - (-20)^3 - 3(-20) - 60 = 161200$

$P(30) = 30^4 - 17(30)^2 - 30^3 - 3(30) - 60 = 767550$

$P(-30) = (-30)^4 - 17(-30)^2 - (-30)^3 - 3(-30) - 60 = 821730$

$P(60) = 60^4 - 17(60)^2 - 60^3 - 3(60) - 60 = 12682560$

$P(-60) = (-60)^4 - 17(-60)^2 - (-60)^3 - 3(-60) - 60 = 13114920$

You can stop if you find the same number of roots as the degree of the Polynomial; otherwise, Keep checking ☺

$\boxed{\#79}$ $P(x) = 2x^3 + x^2 - 16x - 15$

By the rational Root Theorem the possible rational roots are $= \dfrac{\text{divisors of } 15}{\text{divisors of } 2} = \dfrac{\pm1, \pm3, \pm5, \pm15}{\pm1, \pm2} = \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

$P(1) = 2(1)^3 + (1)^2 - 16(1) - 15 = -28$

$P(-1) = 2(-1)^3 + (-1)^2 - 16(-1) - 15 = 0 \leftarrow$ ☺

$P(3) = 2(3)^3 + (3)^2 - 16(3) - 15 = 0 \leftarrow$ ☺ Need one more because the $\deg(P(x)) = 3$

$P(-3) = 2(-3)^3 + (-3)^2 - 16(-3) - 15 = -12$

$P(5) = 2(5)^3 + (5)^2 - 16(5) - 15 = 180$

$P(-5) = 2(-5)^3 + (-5)^2 - 16(-5) - 15 = -160$

$P(\frac{1}{2}) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 16(\frac{1}{2}) - 15 = -22.5$

$P(-\frac{1}{2}) = 2(\frac{-1}{2})^3 + (\frac{-1}{2})^2 - 16(\frac{-1}{2}) - 15 = -7$

$P(\frac{3}{2}) = 2(\frac{3}{2})^3 + (\frac{3}{2})^2 - 16(\frac{3}{2}) - 15 = -30$

$P(\frac{-3}{2}) = 2(\frac{-3}{2})^3 + (\frac{-3}{2})^2 - 16(\frac{-3}{2}) - 15 = 4.5$

$P(\frac{5}{2}) = 2(\frac{5}{2})^3 + (\frac{5}{2})^2 - 16(\frac{5}{2}) - 15 = -17.5$

$P(-\frac{5}{2}) = 2(\frac{-5}{2})^3 + (\frac{-5}{2})^2 - 16(\frac{-5}{2}) - 15 = 0 \leftarrow$ ☺

$\boxed{\#82}$

$$
\begin{array}{r|l}
 & x^3 - 15x^2 + 72x - 108x \\
\hline
x - 9 & x^4 - 24x^3 + 207x^2 - 756x + 972 \\
 & -\,x^4 + 9x^3 \\
\hline
 & -15x^3 + 207x^2 - 756x + 972 \\
 & +\,15x^3 - 135x^2 \\
\hline
 & 72x^2 - 756x + 972 \\
 & -\,72x^2 + 648x \\
\hline
 & -108x + 972 \\
 & +\,108x - 972 \\
\hline
 & 0
\end{array}
$$

$\leftarrow$ Remainder

$P(9) = 9^4 - 24(9)^3 + 207(9)^2 - 756(9) + 972$

$= 6561 - 24(729) + 207(81) - 6804 + 972$

$= 6561 - 17496 + 16767 - 6804 + 972$

$= -10935 + 16767 - 6804 + 972$

$= 5832 - 6804 + 972$

$= -972 + 972$

$= 0$

$\boxed{\#83}$

$$
\begin{array}{r|l}
 & x^3 - 9x^2 + 24x - 16 \\
\hline
x-9 & x^4 - 18x^3 + 105x^2 - 232x + 144 \\
 & \underline{-x^4 + 9x^3} \\
 & \quad -9x^3 + 105x^2 - 232x + 144 \\
 & \quad \underline{+9x^3 - 81x^2} \\
 & \qquad 24x^2 - 232x + 144 \\
 & \qquad \underline{-24x^2 + 216x} \\
 & \qquad\qquad -16x + 144 \\
 & \qquad\qquad \underline{+16x - 144} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

Quotient $= x^3 - 9x^2 + 24x - 16$

Remainder $= 0$

86. Let $P(x) = 4x^3 - 88x^2 + 364x - 408$. Decide which of the statements below are true.

equivalent

(I) $x = 2$ is a zero of $f(x)$.

(II) $x = -2$ is a zero of $f(x)$.

(III) $x + 2$ is a factor of $f(x)$.

(IV) $x - 2$ is a factor of $f(x)$.

(V) The graph of $y = P(x)$ has an x-intercept at $(2, 0)$.

(VI) The graph of $y = P(x)$ has an x-intercept at $(-2, 0)$.

equivalent

Note $P(2) = 4(2)^3 - 88(2)^2 + 364(2) - 408$
$= 4 \cdot 8 - 88 \cdot 4 + 728 - 408$
$= 32 - 352 + 728 - 408$
$= -320 + 728 - 408$
$= 408 - 408$
$= 0 \leftarrow$ So I, IV & V are all true

Possibilities:

Note: $P(-2) = 4(-2)^3 - 88(-2)^2 + 364(-2) - 408$
$= 4 \cdot (-8) - 88 \cdot 4 - 728 - 408$
$= -32 - 352 - 728 - 408$
$= -384 - 728 - 408$
$= -1112 - 408$
$= -1520$
So II, III & VI are all false

~~(a)~~ (I) false; (II) false; (III) false; (IV) true; (V) false; (VI) true

~~(b)~~ (I) true; (II) false; (III) true; (IV) false; (V) true; (VI) false

~~(c)~~ (I) true; (II) false; (III) false; (IV) false; (V) false; (VI) false

(d) (I) true; (II) true; (III) true; (IV) true; (V) true; (VI) true

T $\boxed{\text{(e)}}$ (I) true; (II) false; (III) false; (IV) true; (V) true; (VI) false

87. Which of the following statements are true?

Equivalent so either all true or all false

(I) If $P(x) = 3x^3 + x + 15x^2 + 5$, then $P(5) = 0$.

(II) $(5, 0)$ is an x-intercept on the graph of $y = 3x^3 + x + 15x^2 + 5$.

(III) The remainder of the division problem $\dfrac{3x^3 + x + 15x^2 + 5}{x - 5}$ is zero.

(IV) $(x - 5)$ is a factor of $3x^3 + x + 15x^2 + 5$.

Possibilities:

Note: $P(5) = 3(5)^3 + 5 + 15(5)^2 + 5$
$= 3 \cdot 125 + 5 + 15 \cdot 25 + 5$
$= 375 + 5 + 375 + 5$
$= 380 + 375 + 5$
$= 755 + 5$
$= 760$
So all are false

(a) (I), (II), (III), and (IV) are all true.

(b) Only (III) and (VI) are true.

(c) Only (I) and (III) are true.

(d) Only (I) and (II) are true.

T $\boxed{\text{(e)}}$ None of the statements are true.

88. Find the quotient and the remainder of the divsion problem.

$$\frac{6x^4 + 3x^3 + 2x + 1}{x^2 - 6}$$

Quotient $= 6x^2 + 3x + 36$

Remainder $= 8x + 217$

See Work on Next Page

$\boxed{\#88}$

$$
\begin{array}{r|l}
 & 6x^2+3x+36 \quad \leftarrow \text{Quotient} \\
\hline
x^2+0x-6 & 6x^4+3x^3+0x^2+2x+1 \\
 & -\underline{6x^4-0x^3+36x^2} \\
 & 3x^3+36x^2+2x+1 \\
 & -\underline{3x^3-0x^2+18x} \\
 & 36x^2+20x+1 \\
 & -\underline{36x^2-0x+216} \\
 & 20x+217
\end{array}
$$

Divisor

$20x+217$ ← Stop when the degree here is less then the degree of the Divisor

Remainder

89. Find the quotient and the remainder of the divsion problem.

$$\frac{4x^3 - 3x^2 + 5x - 6}{x - 6}$$

Quotient = $4x^2 + 21x + 131$
Remainder = 780

See Work on Next Page

90. Find the remainder of the divsion problem.

$$\frac{x^4 - 20x^3 + 123x^2 - 180x - 324}{x + 4}$$

Remainder is 3900
and $P(-4) = 3900$

See work on Next page

Let $P(x) = x^4 - 20x^3 + 123x^2 - 180x - 324$. What is $P(-4)$? What is the relationship between $P(-4)$ and the remainder? They are the same.

91. Solve the inequality.

$$\frac{x + 7}{x - 5} \geq 0$$

Test Points	$x+7$	$x-5$	sign
-8	$-$	$-$	$+$
0	$+$	$-$	$-$
6	$+$	$+$	$+$

Possibilities:

(a) $\boxed{(-\infty, -7] \cup (5, \infty)}$

(b) $(-\infty, -7) \cup (5, \infty)$

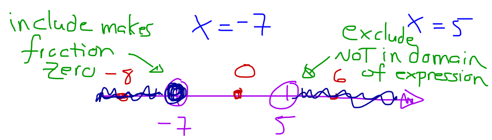
(c) $[-7, 5]$

(d) $(-7, 5)$

(e) $[-7, 5)$

Critical Numbers:

$x + 7 = 0$ or $x - 5 = 0$
$\quad -7 \;\; -7 \qquad\qquad +5 \;\; +5$
$x = -7 \qquad x = 5$



Answer: $(-\infty, -7] \cup (5, \infty)$

92. Solve the inequality.

$$(x + 7)(x - 5) < 0$$

Possibilities:

(a) $(-\infty, \infty)$

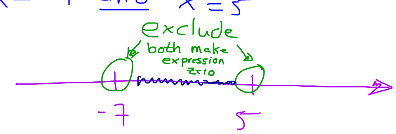
(b) $(-\infty, -7) \cup (5, \infty)$

(c) $[-7, 5]$

(d) $\boxed{(-7, 5)}$

(e) $(-\infty, -7] \cup [5, \infty)$

Critical numbers Same as above
$x = -7$ and $x = 5$



Test Points	$x+7$	$x-5$	sign
-8	$-$	$-$	$+$
0	$+$	$-$	$-$
6	$+$	$+$	$+$

Answer: $(-7, 5)$

$\boxed{\#89}$

$$\begin{array}{r} 4x^2+21x+131 \\ x-6 \overline{\big)\, 4x^3-3x^2+5x-6} \\ -4x^3+24x^2 \\ \hline 21x^2+5x-6 \\ -21x^2+126x \\ \hline 131x-6 \\ -131x+786 \\ \hline 780 \end{array}$$

Quotient $= 4x^2+21x+131$

Remainder $= 780$

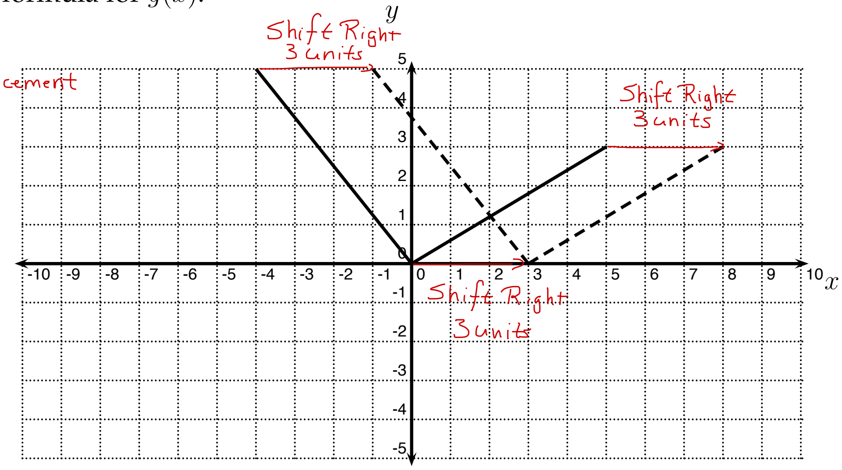
$\boxed{\#90}$

$$\begin{array}{r} x^3-24x^2+219x-1056 \\ x+4 \overline{\big)\, x^4-20x^3+123x^2-180x-324} \\ -x^4-4x^3 \\ \hline -24x^3+123x^2-180x-324 \\ +24x^3+96x^2 \\ \hline 219x^2-180x-324 \\ -219x^2-876x \\ \hline -1056x-324 \\ +1056x+4224 \\ \hline 3900 \end{array}$$

$$\begin{aligned} P(-4) &= (-4)^4-20(-4)^3+123(-4)^2-180(-4)-324 \\ &= 256+20\cdot 64+123\cdot 16+720-324 \\ &= 256+1280+1968+720-324 \\ &= 1536+1968+720-324 \\ &= 3504+720-324 \\ &= 4224-324 \\ &= 3900 \end{aligned}$$

93. In the picture below, the graph of $y = f(x)$ is the solid graph, and the graph of $y = g(x)$ is the dashed graph. Find a formula for $g(x)$.

Shift Right 3 units
algebraically is a replacement of x by $x-3$



Possibilities:

(a) $g(x) = f(x-3)$
(b) $g(x) = f(x) - 3$
(c) $g(x) = f(x) + 3$
(d) $g(x) = -3f(x)$
(e) $g(x) = f(x+3)$

---

94. Find all of the zeros of $P(x) = x^3 - 4x^2 - 12x$.

Possibilities:

(a) 0,6,-2
(b) 0,6,2
(c) 6,-2,8
(d) 0,-6,2
(e) 6,-2,-6

$x^3-4x^2-12x=0$ Factor

$x(x^2-4x-12)=0$

$x(x-6)(x+2)=0$ Zero Product Rule

$x=0$ or $x-6=0$ ($+6$ $+6$) $x=6$ or $x+2=0$ (-2 -2) $x=-2$

Check $x=0$
$0^3-4(0)^2-12(0)=0$ ☺
So $x=0$ is a zero

Check $x=-2$
$(-2)^3-4(-2)^2-12(-2)$
$=-8-16+24$
$=-24+24$
$=0$
So $x=-2$ is a zero

Check $x=6$
$6^3-4(6)^2-12(6)$
$=216-144-72$
$=72-72$
$=0$
So $x=6$ is a zero

---

95. Let $f(x) = 3x - 2$ and $g(x) = x^2$. Find $g(f(x))$.

Possibilities:

(a) $9x^2 - 12x + 4$
(b) $3x^2 - 2$
(c) $3x^3 - 2x^2$
(d) $9x^2 - 4$
(e) $9x^2 + 4$

$g(f(x)) = g(3x-2) = (3x-2)^2 = (3x-2)(3x-2)$
$= 9x^2-6x-6x+4$
$= 9x^2-12x+4$

96. Find all of the zeros of $P(x) = x^3 + 10x^2 + 24x$.

Possibilities:

(a) -4,-6,4

(b) -4,-6,2

(c) 0,-4,-6

(d) 0,-4,6

(e) 0,4,6

$x^3+10x^2+24x=0$ Factor

$x(x^2+10x+24)=0$

$x(x+4)(x+6)=0$ Zero Product Rule

$x=0$ or $x+4=0$ or $x+6=0$

$\quad\quad\quad\; -4\;\; -4 \quad\quad -6\;\; -6$

$\quad\quad\quad\; x=-4 \quad\quad\quad x=-6$

Check $x=0$

$0^3+10(0)^2+24(0)=0$ ☺

so $x=0$ is a zero

Check $x=-4$

$(-4)^3+10(-4)^2+24(-4) = -64+160-96$

$=96-96=0$ ☺

so $x=-4$ is a zero

Check $x=-6$

$(-6)^3+10(-6)^2+24(-6) = -216+360-144$

$=144-144=0$ ☺

So $x=-6$ is a zero

---

97. Determine the end behavior of P(x)=$-2x^{44} + 3x^{33}$.

Highest exponent is even

leading coefficient is Negative

$y \to -\infty$ as $x \to \infty$ &

$y \to -\infty$ as $x \to -\infty$

---

98. Determine the end behavior of P(x)=$2x^{515} + 2x^{514}$.

Highest exponent is odd

leading coefficient is positive

$y \to \infty$ as $x \to \infty$ &

$y \to -\infty$ as $x \to -\infty$

---

99. Which of the following statements are true?

Equivalent so either all true or all false

(I) If $P(x) = 3x^3 + x - 21x^2 - 7$, then $P(7) = 0$.

(II) $(7, 0)$ is an x-intercept on the graph of $y = 3x^3 + x - 21x^2 - 7$.

(III) The remainder of the division problem $\dfrac{3x^3 + x - 21x^2 - 7}{x - 7}$ is zero.

(IV) $(x - 7)$ is a factor of $3x^3 + x - 21x^2 - 7$.

Possibilities:

(a) (I), (II), (III), and (IV) are all true.

(b) Only (I) and (II) are true.

(c) Only (III) and (VI) are true.

(d) Only (I) and (III) are true.

(e) None of the statements are true.

$P(7)=3(7)^3+7-21(7)^2-7$

$=3\cdot 343+7-21\cdot 49-7$

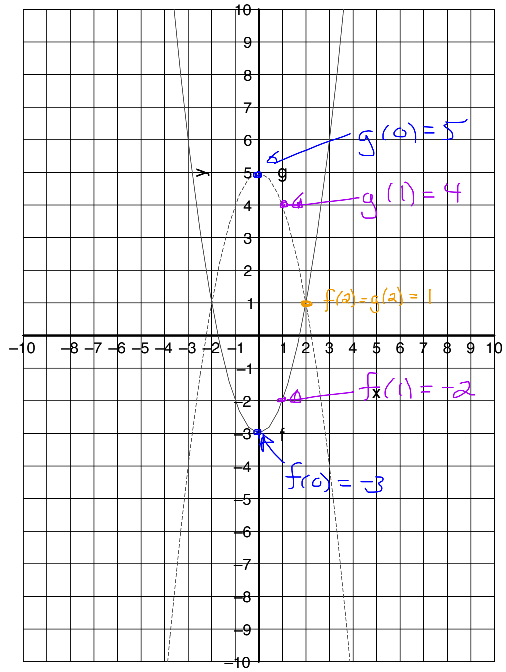
$=1029+7-1029-7$

$=1036-1029-7$

$=7-7$

$=0 \leftarrow$ so all statements are true

100. In the graph below, the solid graph is the graph of $y = f(x)$ and the dashed graph is the graph of $y = g(x)$. Which of the following statements are true?



True
(I) $f(0) < g(0)$

False
(II) $f(1) = g(1)$

False
(III) $f(2) > g(2)$

Possibilities:

(a) **(I)**, **(II)**, and **(III)** are all true.

(b) Only **(II)** is true.

(c) Only **(I)** and **(III)** are true.

(d) None of the statements are true.

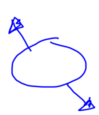
(e) Only **(I)** is true.

---

101. Let $f(x) = -5x^9 + 800x^4 - 6x + 1$. Determine the end behavior of $y = f(x)$.

Highest exponent is odd

Leading coefficient is negative



Possibilities:

(a) $y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

(b) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(c) $y \to \infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(d) $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$

(e) None of the above.

102. Find a formula for the parabola with vertex $(4, -4)$ and that passes through the point $(5, -2)$.

$y = 2(x-4)^2 - 4$ See work on Next Page

103. If the GGMC corporation produces x kilograms of gadgets, then their revenue, in dollars, is given by $R(x) = 100 + 700x - .2x^2$. What is the maximum revenue and how many kilograms of gadgets should be manufactures to obtain this maximum?

Maximum Revenue: \$612,600

Kilograms of Gadgets: 1750

See work on Next page

104. Let $P(x) = x^3 - 3x - 2x^2 + 6$. Find the real zeros of P(x).

See work on following pages

$x = 2, \pm\sqrt{3}$

105. Let $P(x) = 7x^{15} - 2x^7 + 3x^2 + 8$. List all possible rational zeros of $P(x)$ given by the Rational Zeros Theorem (but do not check to see which are actually zeros).

Possibilities:

$\dfrac{\text{Divisors of } 8}{\text{Divisors of } 7} = \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 7} = \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{7}, \pm\frac{2}{7}, \pm\frac{4}{7}, \pm\frac{8}{7}$

(a) $\pm 1, \pm 8, \pm 8/7$

(b) $\pm 1, \pm 2, \pm 4, \pm 8, \pm 7, \pm 7/2, \pm 7/4, \pm 7/8$

(c) $\pm 1, \pm 8, \pm 7/8$

(d) $\pm 1, \pm 1/2, \pm 1/4, \pm 1/8, \pm 7, \pm 7/2, \pm 7/4, \pm 7/8$

(e) $\pm 1, \pm 2, \pm 4, \pm 8, \pm 1/7, \pm 2/7, \pm 4/7, \pm 8/7$

106. Let $f(x) = 3x^2 + 2x + 1$. Find $\dfrac{f(x+h) - f(x)}{h} = 6x + 3h + 2$

See work on following Pages

107. Find an equation for the line that is perpendicular to $y = \frac{5}{6}x + 4$ and passes through the point (4,7).

Point-slope form: $y - y_1 = m(x - x_1)$

$y - 7 = \frac{-6}{5}(x - 4)$

Slope $m = \frac{5}{6}$ $\perp m = \frac{-6}{5}$

108. Solve the inequality.

$$\frac{x-1}{(x-14)^2} > 0$$

Test Point	$x-1$	$(x-14)^2$	sign
0	−	+	−
2	+	+	+
15	+	+	+

Critical Numbers:

$x - 1 = 0$ or $(x-14)^2 = 0$ <-- Square Root both sides or Use Zero Product Rule

$+1 \quad +1$

$x = 1$

Don't include because this will make fraction equal to zero

$x - 14 = 0$

$+14 \quad +14$

$x = 14$

Don't include because this will make the fraction undefined

Possibilities:

(a) $(1, 14)$

(b) $(-\infty, 1) \cup (14, \infty)$

(c) $(-1, 14)$

(d) $(-\infty, 1) \cup (1, \infty)$

(e) $(1, 14) \cup (14, \infty)$

Answer: $(1, 14) \cup (14, \infty)$

#102 Vertex = (4, −4) (h → 4, k → −4)
Point = (5, −2) (x → 5, y → −2)

Standard Form of a quadratic function: $y = a(x-h)^2 + k$

I like to call this "vertex form"

Vertex $h = 4, k = -4$ Simplify

$y = a(x-4)^2 + (-4)$

* only need the value for "a"

$y = a(x-4)^2 - 4$

Point $x = 5, y = -2$ Simplify

$-2 = a(5-4)^2 - 4$

$-2 = a(1)^2 - 4$

$-2 = a - 4$ Add 4

$-2 + 4 = a - 4 + 4$ Simplify

$2 = a$

$a = 2$

$\boxed{y = 2(x-4)^2 - 4}$

#103 $R(x) = 100 + 700x - 0.2x^2$

parabola opens downward because $a = -0.250$

maximum point

Vertex

1st Compute: $\dfrac{-b}{2a} = \dfrac{-700}{2(-0.2)} = \dfrac{-700}{-0.4} = 1750$ ← where the maximum occurs

2nd Compute $R\left(\frac{-b}{2a}\right) = R(1750) = 100 + 700\cdot(1750) - 0.2(1750)^2$

$= 100 + 1225000 - 0.2 \cdot 3062500$

$= 1225100 - 612500$

$= 612600$

Maximum Revenue

#104 $P(x) = x^3 - 3x - 2x^2 + 6$

By the ration root theorem the possible rational roots are $= \dfrac{\text{Divisors of } 6}{\text{Divisors of } 1} = \dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 1^3 - 3(1) - 2(1)^2 + 6 = 2$

$P(-1) = (-1)^3 - 3(-1) - 2(-1)^2 + 6 = 6$

$P(2) = 2^3 - 3(2) - 2(2)^2 + 6 = 0$ ← with this we can find the other real roots

Since 2 is a root $(x-2)$ is a factor of $P(x) = x^3 - 3x - 2x^2 + 6$

See Next Page

$\boxed{\#104 \text{ continued}}$

$$\begin{array}{r} x^2 - 3 \\ x-2 \overline{\big) x^3 - 2x^2 - 3x + 6} \\ \underline{-x^3 + 2x^2 } \\ -3x + 6 \\ \underline{+3x - 6} \\ 0 \end{array}$$

Note I ordered terms from highest exponent to lowest.

Consequently $P(x) = x^3 - 2x^2 - 3x + 6 = (x^2-3)(x-2) + 0$
$= (x^2-3)(x-2)$

So to find when $P(x) = 0$
we need $(x^2-3)(x-2) = 0$ Zero Product Rule

$x^2 - 3 = 0$
$+3 \quad +3$
$x^2 = 3$
$x = \pm\sqrt{3}$

$x - 2 = 0$
$+2 \quad +2$
$x = 2$

Check $x=2$
$P(2) = 2^3 - 3(2) - 2(2)^2 + 6$
$= \cancel{8} - \cancel{6} - \cancel{8} + \cancel{6}$
$= 0$
So $x=2$ is a zero

Check $x = \sqrt{3}$
$P(\sqrt{3}) = (\sqrt{3})^3 - 3\sqrt{3} - 2(\sqrt{3})^2 + 6$
$= 3\sqrt{3} - 3\sqrt{3} - 2 \cdot 3 + 6$
$= \cancel{3\sqrt{3}} - \cancel{3\sqrt{3}} - \cancel{6} + \cancel{6}$
$= 0$
So $x = \sqrt{3}$ is a zero

Check $x = -\sqrt{3}$
$P(-\sqrt{3}) = (-\sqrt{3})^3 - 3(-\sqrt{3}) - 2(-\sqrt{3})^2 + 6$
$= -3\sqrt{3} + 3\sqrt{3} - 2 \cdot 3 + 6$
$= \cancel{-3\sqrt{3}} + \cancel{3\sqrt{3}} \cancel{-6} + \cancel{6}$
$= 0$
So $x = -\sqrt{3}$ is a zero

$\boxed{\#106}$ 1st Compute $f(x+h) = 3(x+h)^2 + 2(x+h) + 1$

$= 3(x+h)(x+h) + 2x + 2h + 1$

$= 3(x^2 + xh + xh + h^2) + 2x + 2h + 1$

$= 3(x^2 + 2xh + h^2) + 2x + 2h + 1$

$= 3x^2 + 6xh + 3h^2 + 2x + 2h + 1$

2nd Compute $\frac{f(x+h) - f(x)}{h} = \frac{(3x^2 + 6xh + 3h^2 + 2x + 2h + 1) - (3x^2 + 2x + 1)}{h}$

$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{+2x} + 2h \cancel{+1} \cancel{-3x^2} \cancel{-2x} \cancel{-1}}{h}$

$= \frac{6xh + 3h^2 + 2h}{h} = \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}}$

$= \boxed{6x + 3h + 2}$

109. Which of the following equations are linear equations?

(I) $y - 2 = x + 3$ — Yes

(II) $y = (\sqrt{2})x$ — Yes

(III) $y = \sqrt{2x}$

$y = \sqrt{2}\sqrt{x}$

$y = \sqrt{2}\, x^{\frac{1}{2}}$

NO

Possibilities:

(a) All of the equations are linear.

(b) None of the equations are linear.

(c) Only equation **(I)** is linear.

$\boxed{\text{(d) Only equations (I) and (II) are linear}}$

(e) Only equations **(II)** and **(III)** are linear

---

110. Which of the following are equations for the line through the points $P(-2, 4)$ and $Q(5, 6)$?

(I) $y - 6 = \dfrac{2}{7}(x - 5)$ ✓ **(II)** $y = \dfrac{2}{7}x - 6$ ✗

(III) $y = \dfrac{2}{7}(x + 2) + 4$ ✓ **(IV)** $y + 4 = \dfrac{2}{7}(x - 2)$ ✗

Possibilities:

(a) Only **(II)** and **(IV)**.

$\boxed{\text{(b) Only (I) and (III).}}$

(c) Only **(I)**.

(d) Only **(II)**.

(e) Only **(IV)**.

(I)
$4 - 6 \overset{?}{=} \frac{2}{7}(-2-5)$
$-2 \overset{?}{=} \frac{2}{7}(-7)$
$-2 = -2$ so P is on the line
$6 - 6 \overset{?}{=} \frac{2}{7}(5-5)$
$0 \overset{?}{=} \frac{2}{7}(0)$
$0 = 0$ so Q is on the line

(II)
$4 \overset{?}{=} \frac{2}{7}(-2) - 6$
$4 \overset{?}{=} \frac{-4}{7} - \frac{6}{1}\cdot\frac{7}{7}$
$4 \overset{?}{=} -\frac{4}{7} - \frac{42}{7}$
$4 \neq \frac{46}{7}$ so P is not on the line

(III)
$4 \overset{?}{=} \frac{2}{7}(-2+2) + 4$
$4 \overset{?}{=} \frac{2}{7}(0) + 4$
$4 \overset{?}{=} 0 + 4$
$4 = 4$ so P is on the line
$6 \overset{?}{=} \frac{2}{7}(5+2) + 4$
$6 \overset{?}{=} \frac{2}{7}\cdot 7 + 4$
$6 \overset{?}{=} 2 + 4$
$6 = 6$ so Q is on the line

(IV)
$4 + 4 \overset{?}{=} \frac{2}{7}(-2-2)$
$8 \overset{?}{=} \frac{2}{7}(-4)$
$8 \neq \frac{-8}{7}$ so P is not on the line

---

111. Let $f(x) = x^2 + 5x$. Find the average rate of change of $f(x)$ from $\underset{a}{x = a}$ to $\underset{b}{x = a + h}$. Assume $h \neq 0$.

Possibilities:

(a) $\dfrac{h^2 + 5h}{h}$

(b) $\dfrac{2ah + h^2 + 10a + 5h}{h}$

(c) $-2a - h - 5$

$\boxed{\text{(d) } 2a + h + 5}$

(e) 1

1st Compute $f(a+h) = (a+h)^2 + 5(a+h) = (a+h)(a+h) + 5a + 5h$
$= a^2 + ah + ha + h^2 + 5a + 5h = a^2 + 2ah + h^2 + 5a + 5h$

and $f(a) = a^2 + 5a$

Average Rate of Change $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{f(a+h) - f(a)}{a + h - a} = \dfrac{(a^2 + 2ah + h^2 + 5a + 5h) - (a^2 + 5a)}{h}$

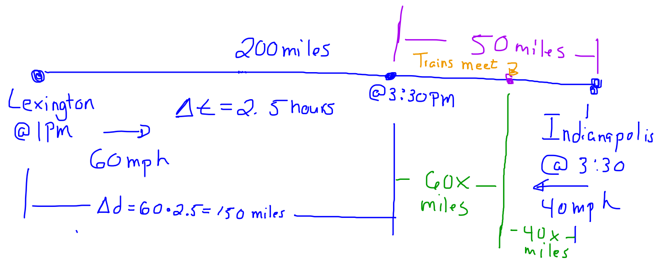
$= \dfrac{\cancel{a^2} + 2ah + h^2 + \cancel{5a} + 5h - \cancel{a^2} - \cancel{5a}}{h} = \dfrac{2ah + h^2 + 5h}{h} = \dfrac{\cancel{h}(2a + h + 5)}{\cancel{h}}$

$= \boxed{2a + h + 5}$

112. A train leaves Lexington for Indianapolis, 200 miles away, at 1:00 PM and averages 60 miles per hour. A second train travelling on an adjacent track leaves Indianapolis for Lexington at 3:30 PM and averages 40 miles per hour. At what time will the trains meet? (Round to the nearest minute.)

Possibilities: Let x be the number of hours from 3:30pm until the two trains meet

(a) 5:00PM
(b) 4:00PM
(c) 5:30PM
(d) 4:30PM
(e) 6:00PM



$60x + 40x = 50$

$\frac{100x}{100} = \frac{50}{100}$

$x = \frac{1}{2}$ hour from 3:30PM is $\boxed{4\text{PM}}$

113. Approximate the solution to $8x^3 + 24x^2 + 24x + 8 = 9$.

Possibilities:

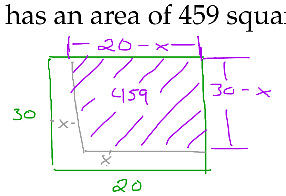
(a) $x \approx 0.0200$
(b) $x \approx 0.0139$
(c) $x \approx 0.0400$
(d) $x \approx -0.3920$
(e) $x \approx 9.0000$

114. A corner lot has dimensions 30 yards by 20 yards. The city plans to take a strip of uniform width along the two sides bordering the streets to widen these roads. How wide should the strip be if the remainder of the lot has an area of 459 square yards?

Let "x" be the width of the uniform strip removed from the corner lot. Note $0 \le x \le 20$

Possibilities:

(a) 30 yards
(b) 3 yards
(c) 459 yards
(d) 1 yard
(e) 10 yards



$(20-x)(30-x) = 459$

$600 - 20x - 30x + x^2 = 459$

$x^2 - 50x + 600 = 459$

$x^2 - 50x + 600 - 459 = 459$

$x^2 - 50x + 141 = 0$

$x^2 - 3x - 47x + 141 = 0$

$x(x-3) - 47(x-3) = 0$

$(x-47)(x-3) = 0$

$x - 47 = 0$
$+47 \quad +47$
$x = 47$
Impossible because $0 \le x \le 20$

or

$x - 3 = 0$
$+3 \quad +3$
$\boxed{x = 3}$

115. Which of the following windows is an appropriate viewing window for $y = 18x - 3x^2$?

Possibilities:

(a) $-5 \le x \le 25, 0 \le y \le 20$
(b) $-10 \le x \le 10, -50 \le y \le 50$
(c) $-30 \le x \le 15, -100 \le y \le 250$
(d) $-10 \le x \le 10, -10 \le y \le 10$
(e) None of the above windows gives a complete graph.

Short Answer Questions

Clearly write your final answer on the front page of the exam.

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116. Let $P(x) = 2x^7 - 493x^5 + 1050$. Find the y-intercept of the graph of $y = P(x)$.

$\underline{\text{y-intercept:}}$ $x = 0$

$P(0) = 2(0)^7 - 493(0)^5 + 1050 = 1050$

so the y-intercept is $\boxed{(0, 1050)}$

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117. A manufacturer finds that the revenue generated by selling x gadgets is given by the function $R(x) = 340x - .8x^2$, where the revenue $R(x)$ is is measured in dollars. What is the maximum revenue?

$\frac{-b}{2a} = \frac{-340}{2(-0.8)} = \frac{340}{1.6} = 212.5$

$R(212.5) = 340(212.5) - 0.8(212.5)^2 = 72250 - 0.8(45156.25)$

$= 72250 - 36125 = \boxed{\$36125}$

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118. Find the **remainder** of the divsion problem.

$$\frac{x^4 + 7x^3 - 9x^2 - 115x - 100}{x - 1}$$

Let $P(x) = x^4 + 7x^3 - 9x^2 - 115x - 100$

then the remainder is the same

as $P(1) = 1^4 + 7(1)^3 - 9(1)^2 - 115(1) - 100$

$= 1 + 7 - 9 - 115 - 100$

$= 8 - 9 - 115 - 100$

$= -1 - 115 - 100$

$= -116 - 100$

$= \boxed{-216}$

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119. Let $f(x) = 2x^2 - 36x + 155$. Answer the following questions about the graph of $y = f(x)$.

positive so minimum

(a) Does the graph of $y = f(x)$ have a minimum or a maximum at its vertex? $\boxed{\text{minimum}}$

(b) What is the vertex of $y = f(x)$?

$\frac{-b}{2a} = \frac{-(-36)}{2(2)} = \frac{36}{4} = 9$

$\boxed{\text{Vertex} = (9, -7)}$

$f(9) = 2(9)^2 - 36(9) + 155 = 2 \cdot 81 - 324 + 155$

$= 162 - 324 + 155 = -162 + 155 = -7$
