MA 109 — College Algebra SPRING 2007
THIRD MIDTERM 04/11/2007

Name: Key **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

Part I consists of 12 multiple choice questions worth 5 points each. Record your answers on this page by filling in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] [b] [c] [d] [e]

Do not circle answers on this page, but please do circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

Part II consists of 4 partial credit problems worth a total of 40 points. Write your answer and show ALL your work on the page on which the question appears.

GOOD LUCK!

1. [a] [b] [c] [d] [e]

2. [a] [b] [c] [d] [e]

3. [a] [b] [c] [d] [e]

4. [a] [b] [c] [d] [e]

5. [a] [b] [c] [d] [e]

6. [a] [b] [c] [d] [e]

7. [a] [b] [c] [d] [e]

8. [a] [b] [c] [d] [e]

9. [a] [b] [c] [d] [e]

10. [a] [b] [c] [d] [e]

11. [a] [b] [c] [d] [e]

12. [a] [b] [c] [d] [e]

For grading use:

Question	Score	Total
1-12		60
13		10
14		10
15		10
16		10
Total	(out of 100 pts)	100

Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table:

Section #	Instructor	Lectures
002	W. Guo	MWF 9:00am-9:50am, BE 213
003	G. Butcher	MWF 10:00am-10:50am, CB 337
004	A. Bagchi Misra	MWF 10:00am-10:50am, FB 213
005	G. Butcher	MWF 11:00am-11:50am, CB 335
006	A. Corso	MWF 12:00pm-12:50pm, CB 114
008	W. Guo	MWF 1:00pm-1:50pm, CB 335
009	L. Roberson	MWF 2:00pm-2:50pm, CB 337
010	A. Bagchi Misra	MWF 3:00pm-3:50pm, CB 335
011	L. Roberson	MWF 3:00pm-3:50pm, CB 337
013	S. Nanwani	TR 8:00am-9:15am, CB 335
019	P. Zhang	TR 12:30pm-1:45pm, BS 108
020	S. Nanwani	TR 2:00pm-3:15pm, CB 333
021	Y. Li	TR 2:00pm-3:15pm, CB 347
022	Y. Li	TR 3:30pm-4:45pm, CB 335
023	P. Zhang	TR 3:30pm-4:45pm, CB 339
401	C. Norman	TR 6:00pm-7:15pm, CP 201
403	E. Stokes	TR 6:00pm-7:15pm, CB 337
404	E. Stokes	TR 7:30pm-8:45pm, CB 337

PART I

Multiple Choice Questions (5 points each)

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Compute the average rate of change of the function

$$f(x) = 2x^2 - 4x$$

between $x = -2$ and $x = 4$.

$f(-2) = 2(-2)^2 - 4(-2) = 16$

$f(4) = 2(4)^2 - 4(4) = 16$

rate $= \dfrac{16-16}{4-(-2)} = 0$

Possibilities:

(a) 5

(b) 0

(c) 1

(d) -5

(e) 6

---

2. Let $g(x) = -5x + 3$ and let $h \neq 0$. Then the difference quotient

$$\frac{g(a+h) - g(a)}{h}$$

is given by one of the following expressions.

$g(a) = -5(a) + 3$

$g(a+h) = -5(a+h) + 3$

$$\frac{-5a - 5h + 3 - (-5a + 3)}{h}$$

$$\frac{-5a - 5h + 3 + 5a - 3}{h} = -5$$

Possibilities:

(a) -5

(b) $5h$

(c) 1

(d) $-10a + h$

(e) 0

3. State how the graph of the function

$$h(x) = \frac{1}{x-4} - 1$$

can be obtained from the graph of $f(x) = \frac{1}{x}$. asymtopes are $Y=0$ $X=0$

vertical asymtopes for $\frac{1}{x-4}-1$ is at $X=4$

horizontal at $Y=-1$

Possibilities:

(a) The graph of $f(x)$ is translated 4 units to the left and 1 unit down

(b) The graph of $f(x)$ is translated 4 units to the right and 1 unit down

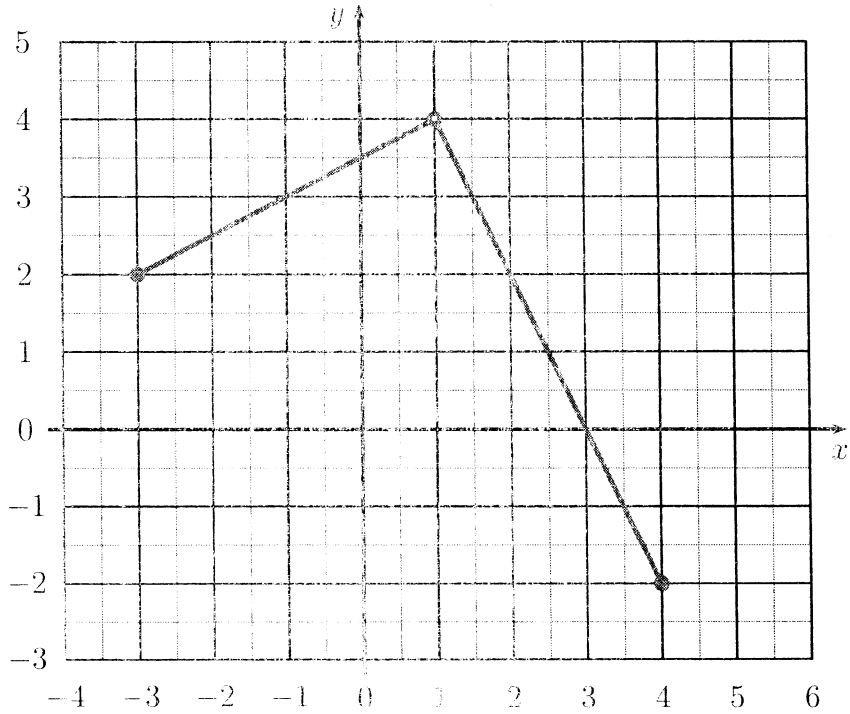
(c) The graph of $f(x)$ is reflected about the x-axis and translated 4 units to the right

(d) The graph of $f(x)$ is translated 4 units down

(e) The graph of $f(x)$ is translated 1 unit to the left and 4 units down

---

4. Specify the domain and range of the function graphed below



Possibilities:

(a) domain: $[-3, 4]$; range: $[-2, 4)$

(b) domain: $[-3, 1) \cup (1, 4]$; range: $[-2, 4)$

(c) domain: $[-2, 4]$; range: $[-3, 1) \cup (1, 4]$

(d) domain: $[-3, 4]$; range: $[-2, 2]$

(e) domain: $[-2, 4)$; range: $[-3, 4]$

Domain is set of all X values

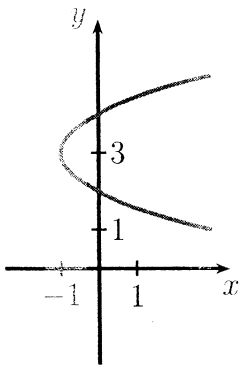
$[-3, 1) \cup (1, 4]$

rang is the set of all Y values

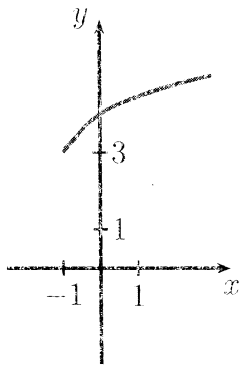
$[2, 4)$

5. Find the graph of the function $y = \sqrt{x-1} + 3$.

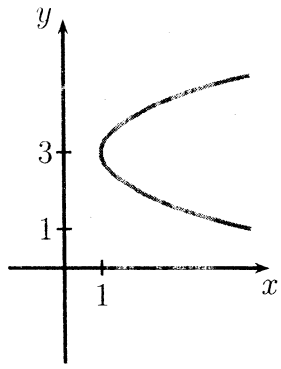
(A)



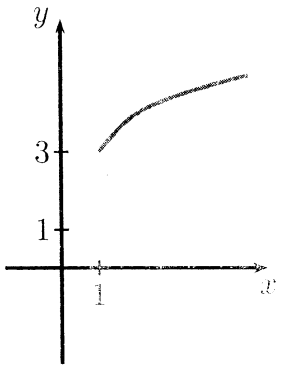
(B)



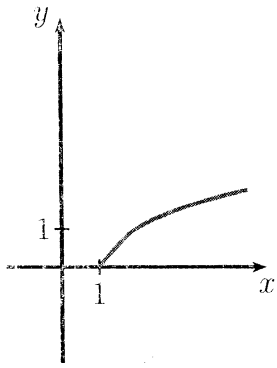
(C)



(D)



(E)



$\sqrt{x-1}$ tells us to translate right 1

$+3$ tells us to translate up 3

So D is correct since we only consider the positive values when we take the square root.

Possibilities:

(a) (A)
(b) (B)
(c) (C)
(d) (D)
(e) (E)

---

6. Find the x- and y-intercepts of the function $y = x^3 - x$.

$y = x(x^2-1)$

$y = x(x-1)(x+1)$

x-intercepts at $-1, 0, 1$

since $y = 0^3 - 0 = 0$

y-intercept at $y = 0$

Possibilities:

(a) x-intercepts: $-3, 0, 3$ and y-intercept: 0
(b) x-intercepts: 0 and y-intercept: 0
(c) x-intercepts: $1, 3$ and y-intercept: 3
(d) no x-intercept and no y-intercept
(e) x-intercepts: $-1, 0, 1$ and y-intercept: 0

7. Determine the coordinates of the point P where the minimum value of the function

$$f(x) = x^2 - 8x + 7$$

occurs.

X-coordinate at $\frac{-b}{2 \cdot a}$ $\quad \frac{-(-8)}{2} = 4$

Y-coordinate $f(4) = (4)^2 - 8(4) + 7 = -9$

So $P = (4, -9)$

Possibilities:

(a) $P = (-8, 7)$

(b) $P = (-4, 9)$

(c) $P = (-4, -9)$

(d) $P = (4, -9)$

(e) $P = (-4, 7)$

---

8. Find the equation for the parabola with vertex $(-2, 1)$ and passing through the point $(1, 19)$.

equation for a parabola:

$(y - y_1) = a(x - x_1)^2$ where (x_1, y_1) is the vertex.

$y - 1 = a(x+2)^2$

$y = a(x^2 + 4x + 4) + 1$

Solve for a using the point $(1, 19)$

$19 = a(1^2 + 4 \cdot 1 + 4) + 1$

$19 - 1 = a \cdot 9$

$\frac{18}{9} = 9 = 2$

$y = 2(x+2)^2 + 1$

Possibilities:

(a) $y = 2(x-2)^2 + 1$

(b) $y = \frac{1}{19}(x+2)^2 + 1$

(c) $y = 2(x+2)^2 + 1$

(d) $y = -2(x-1)^2 + 19$

(e) $y = -2x^2 + x + 19$

9. Find the horizontal and vertical asymptotes of the function

$$y = \frac{x^2 - 4}{9x^2 - 1}$$

horizontal asymptote(s) found by the leading coeficients

$y = \frac{1 \cdot x^2 - 4}{9 \cdot x^2 - 1}$ $\quad y = \frac{1}{9}$

vertical asymptote(s) when the denominator equals zero

$9x^2 - 1 = 0$ $\quad x^2 = \frac{1}{9}$

$9x^2 = 1$ $\quad x = \pm\frac{1}{3}$

Possibilities:

(a) horizontal asymptote: none vertical asymptotes: $x = -\frac{1}{3}$ and $x = \frac{1}{3}$

(b) horizontal asymptote: $y = \frac{1}{9}$ vertical asymptotes: none

(c) horizontal asymptote: $y = \frac{1}{9}$ vertical asymptotes: $x = -\frac{1}{3}$ and $x = \frac{1}{3}$

(d) horizontal asymptote: $y = 9$ vertical asymptotes: $x = -1$ and $x = 4$

(e) horizontal asymptote: $y = 0$ vertical asymptotes: $x = -2$ and $x = 2$

---

10. The domain of the function

$$f(x) = \sqrt{(4 - x)(x + 7)}$$

is given by one of the following interval(s).

Square root cannot be negative so we need to determine when $(4-x)(x+7) < 0$

We pick 3 points around the zeros. Zeros at $x = -7, 4$

$x = -8$

$x = 0$

$x = 5$

$(4-(-8))(-8+7) < 0$ so #s less than -7 are not in the domain

$(4-0)(0+7) > 0$ so #s greater than -7 but 4≤ are in the domain

$(4-5)(5+7) < 0$ so #s greater than 4 are not in the domain.

So $-7 \leq x \leq 4$

Possibilities:

(a) $x \leq -7,\ x \geq 4$

(b) $x \geq 0$

(c) $-4 \leq x \leq 7$

(d) $x \neq -7$ and $x \neq 4$

(e) $-7 \leq x \leq 4$

---

11. Which of the following is a complete list of all possible rational zeros of the polynomial

$$P(x) = 2x^3 - 3x^2 + 7x - 11$$

as given by the Rational Zeros Theorem?

$\pm\frac{11}{1}, \pm\frac{11}{2}, \pm\frac{1}{1} \pm\frac{1}{2}$

Possibilities:

(a) $\pm\frac{1}{11}, \pm 11$

(b) $\pm 11, \pm 22$

(c) $\pm\frac{1}{2}, \pm 1, \pm\frac{11}{2}, \pm 11$

(d) $\pm 2, \pm 3, \pm 7, \pm 11$

(e) ± 1

---

12. The following synthetic division table

root $P(x)$

3	2	0	−20	5
		6	18	−6
	2	6	−2	−1

corresponds to one of the following problems. Which one is the correct one?

3 is the root so $D(x) = x - 3$

(a) $P(x) = 2x^3 + 0x^2 - 20x + 5$

Possibilities:

(a) Divide $P(x) = 2x^3 - 20x + 5$ by $D(x) = x - 3$

(b) Divide $P(x) = 2x^3 - 20x^2 + 5$ by $D(x) = x^2 - 3$

(c) Divide $P(x) = 2x^3 - 20x + 5$ by $D(x) = x + 3$

(d) Divide $P(x) = 2x^3 - 20x^2 + 5x$ by $D(x) = x - 3$

(e) Divide $P(x) = 2x^3 + 6x^2 - 2x - 1$ by $D(x) = x + 3$

PART II

Partial Credit Questions (10 points each)

Show all your work and put your final answer in the space provided.
Use the backs of the test sheets if you need more workspace.
No credit will be given for a correct answer without showing how it was obtained.
You will receive no credit if the answer is not in the space provided and no partial credit for a wrong answer if you do not show your work.

13. Consider the polynomial

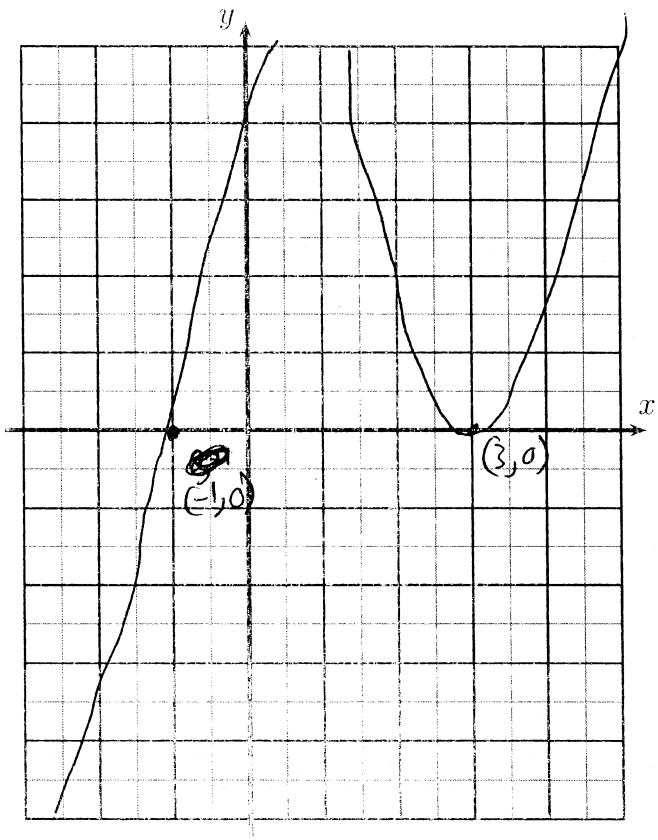
$$P(x) = (x+1)(x-3)^2$$

(*a*) Use the factored form of $P(x)$ to find the zeros. $P(x)=0$ when $x=-1$ and $x=3$

(*b*) Determine the end behavior and then sketch the graph of

$$y = (x+1)(x-3)^2$$

in the space provided below.



Zeros: $x=-1$ & $x=+3$

End Behavior: $x \to \infty: P(x) \to \infty$ $x \to -\infty$ $P(x) \to -\infty$

pts: /10

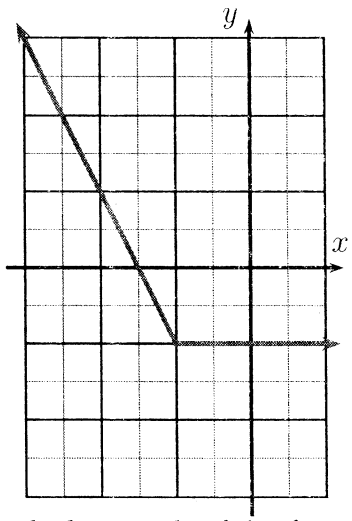
14. (*a*) Factor the polynomial $P(x) = x^3 + x^2 - 6x$ and use the factored form to find its zeros.

$P(x) = x(x^2+x-6)$

$= x(x+3)(x-2)$

Answer: $x = -3, 0, 2$

(*b*) The graph of the function f is given below

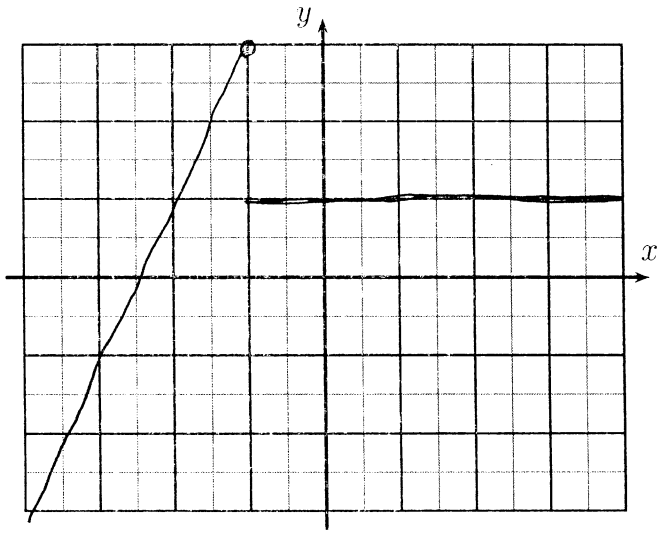


$$f(x) = \begin{cases} -2x - 6 & x < -2 \\ -2 & x \geq -2 \end{cases}$$

Carefully sketch the graph of the function $y = -f(x+2)$.

$$f(x+2) = \begin{cases} -2x-10 & x < -2 \\ -2 & x \geq -2 \end{cases}$$

$$-f(x+2) = \begin{cases} 2x+10 & x < -2 \\ 2 & x \geq -2 \end{cases}$$



pts: /10

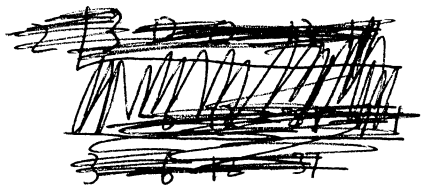
15. (a) Let $P(x) = x^4 - x^3 + 5x + 3$ and $D(x) = x^2 - 1$. Use the long division algorithm to find polynomials $Q(x)$ and $R(x)$ satisfying $P(x) = Q(x) \cdot D(x) + R(x)$.

$$
\begin{array}{r}
x^2 - x + 1 \\
x^2 - 1 \;\overline{)\; x^4 - x^3 + 5x + 3} \\
-(x^4 \quad + x^2) \\
\hline
-x^3 + x^2 + 5x \\
-(-x^3 \quad + x) \\
\hline
x^2 + 4x + 3 \\
-(x^2 \quad - 1) \\
\hline
4x + 4
\end{array}
$$

$Q(x)$: $x^2 - x + 1$

$R(x)$: $4x + 4$

(b) Use synthetic division to divide $P(x) = 3x^3 - 13x + 17$ by $D(x) = x + 2$ to find polynomials $Q(x)$ and $R(x)$ satisfying $P(x) = Q(x) \cdot D(x) + R(x)$.
Use $Q(x)$, $R(x)$ and the Remainder Theorem to evaluate $P(-2)$.



$$
\begin{array}{r|cccc}
-2 & 3 & 0 & -13 & 17 \\
 & & -6 & 12 & 2 \\
\hline
 & 3 & -6 & -1 & 19
\end{array}
$$

$Q(x) = 3x^2 - 6x - 1$

$P(x) = (x+2)(3x^2 - 6x - 1) + 19$

$P(-2) = 19$

$R(x) = 19$

$P(-2) = 19$

pts: /10

16. (a) The table below gives the population in a small rural community for the period 1950-1960. Figures shown are for January 1 in each year.

Year	Population
1950	624
1951	856
1952	1,336
1953	1,578
1954	1,591
1955	1,483
1956	994
1957	826
1958	801
1959	745
1960	790

What was the average rate of change of the population between 1954 and 1958?
For what period of time was the population decreasing?

Average rate of change = $\frac{\text{change in Population}}{\text{change in time}}$

$R = \frac{-790}{4}$

average rate of change: $-790/4$

population decreasing during: 1954 → 1959

(b) Let $f(x) = x^2$. Find and simplify the expression:

$$f(2+h) - f(2-h).$$

$(2+h)^2 - (2-h)^2$

$4 + 4h + h^2 - (4 - 4h + h^2)$

$4 + 4h + h^2 - 4 + 4h - h^2$

$\boxed{8h}$

Answer: $8h$

pts: /10