Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam.
No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

\[ \text{a} \quad \text{b} \quad \text{c} \quad \text{d} \quad \text{e} \]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e
9. a b c d e
10. a b c d e
11. a b c d e
12. a b c d e
13. a b c d e
14. a b c d e
15. a b c d e
16. a b c d e
17. a b c d e
18. a b c d e
19. a b c d e
20. a b c d e

For grading use:

<table>
<thead>
<tr>
<th>Number Correct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(out of 20 problems)</td>
<td>(out of 100 points)</td>
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</table>
Multiple Choice Questions
Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

1. Use the substitution method to find all solutions of the system of equations.
   \[
   \begin{align*}
   x^2 + y &= 15 \\
   x - y + 3 &= 0
   \end{align*}
   \]
   Possibilities:
   (a) \((x = 4, y = -1)\) and \((x = -4, y = 7)\)
   (b) \((x = -4, y = -1)\) and \((x = 3, y = 6)\)
   (c) \((x = 3, y = 6)\) only
   (d) \((x = -4, y = -1)\) only
   (e) No real solutions

2. Suppose you are solving the system of equations below using the substitution method. You solve for \(y\) in the first equation and substitute it into the second equation. What equation must you solve then?
   \[
   \begin{align*}
   4x^9 + y &= 2 \\
   9x^3 + 5y &= 6
   \end{align*}
   \]
   Possibilities:
   (a) \(9x^3 + 5(2 - 4x^8) = 6\)
   (b) \(9\left(\frac{\sqrt{2} - 4x^8}{3}\right) + \frac{x}{y} = 6\)
   (c) \(9(2 - 4x^8)^3 + 5y = 6\)
   (d) \(9x^3 + 5\left(\frac{\sqrt{2}}{y} - y\right) = 6\)
   (e) \(9x^3 + 5\left(\frac{\sqrt{2} - 4x^8}{3}\right) = 6\)
3. Use the elimination method to solve the system. The multiple choice problem only asks you for $y$.

Possibilities:

(a) Every solution has $y = \frac{4}{17}$
(b) Every solution has $y = 3$
(c) Every solution has $y = 4$
(d) Every solution has $y = \frac{4}{13}$
(e) Every solution has $y = -13$

* this question does not require you to also solve for $x$!

4. Use the elimination method to find all solutions of the system of equations.

Possibilities:

(a) $(x = 4, y = 3)$ only
(b) $(x = -9, y = 7)$ and $(x = -8, y = 7)$
(c) $(x = 4, y = 3)$ and $(x = -4, y = -3)$
(d) $(x = 4, y = 3), (x = -4, y = 3), (x = 4, y = -3)$, and $(x = -4, y = -3)$
(e) $(x = 9, y = 7)$ and $(x = 8, y = 7)$

* use solutions for $x$ to solve for $y$ also
5. Use the elimination method to find all solutions of the system of equations.

**easiest to eliminate y-variable 1st since one is a multiple of the other**

\[
\begin{align*}
\frac{37}{x} + \frac{14}{y} &= 113 \\
\frac{17}{x} + \frac{7}{y} &= 54 \\
\frac{37}{x} + \frac{14}{y} &= 113 \\
\frac{-34}{x} + \frac{-14}{y} &= -108 \\
2\left(\frac{3}{x} = \frac{3}{5}\right)
\end{align*}
\]

Possibilities:
(a) \((x = 37, y = 14)\) and \((x = 17, y = 7)\)
(b) \(\left(\frac{3}{5}, y = \frac{3}{11}\right)\) and \(\left(\frac{-3}{5}, y = -\frac{3}{11}\right)\)
(c) \(\left(\frac{3}{5}, y = \frac{3}{11}\right)\) only
(d) \((x = -37, y = 14)\) and \((x = -17, y = 7)\)
(e) \((x = 37, y = 14)\), \((x = -37, y = 14)\), \((x = 17, y = 7)\), and \((x = -17, y = -7)\)

*Now substitute \(x\)-solution back into one of original equations to solve for \(y\)

\[\frac{7}{y} = \frac{162-85}{3} = \frac{77}{3}\]
\[\left(\frac{7}{y} = \frac{77}{3}\right)\]
\[\frac{7}{y} = 54-\frac{85}{3} = \frac{21}{y} = 77\]

6. The graph of two equations is shown below. Determine the number of solutions for the system of equations.

**solutions to systems are points of intersection of graphs!!**

Possibilities:
(a) 3
(b) 1
(c) 2
(d) 0
(e) 4
7. Use graphical approximation (a root finder or an intersection finder) to find a solution of the equation in the given interval. (Round your answer to four decimal places.)

\[ x^5 + 4 = 8x^4; \quad (-\infty, 0] \]

Possibilities:
(a) \( x = -0.8250 \)
(b) \( x = -0.8239 \)
(c) \( x = -0.8228 \)
(d) \( x = -0.8217 \)
(e) \( x = -0.8206 \)

* graph 2 equations

* look for intersection point to the left of y-axis ... this is the interval \((-\infty, 0]\)

OR * graph 1 equation

\[ y = x^5 - 8x^4 + 4 \]

* look for x-intercept (when \( y = 0 \)) that is to the left of y-axis

* then record x-value

8. A corner lot has dimensions 40 by 33 yards. The city plans to take a strip of uniform width along the two sides bordering the streets to widen these roads. How wide should the strip be if the remainder of the lot is to have an area of 980 square yards?

Which equation should you solve in order to find the answer? The variable \( x \) represents the width of the strip in yards.

Possibilities:
(a) \((40)(33) - x^2 = 980\)
(b) \((40 - x)(33 - x) = 980\)
(c) \(x^2 = 980\)
(d) \(x = 1320 - 980\)
(e) \((40)(33) = x\)

\[ \text{new area} = 980 \]
\[ \text{l.w} = 980 \]
\[ (40-x)(33-x) = 980 \]
9. You have already invested $400 in a stock with an annual return of 10%. How much of an additional $1,400 should be invested at 20% and how much at 5% so that the total return on the entire $1,800 is 15%?

What equations should be solved if \( x \) is the amount of money invested at 20% and \( y \) is the amount of money invested at 5%?

Possibilities:

(a) \[
\begin{align*}
400 + x + y &= 1800 \\
0.10(400) + 0.20x + 0.05y &= 0.15(1800)
\end{align*}
\]

(b) \[
\begin{align*}
0.15 + x + y &= 1800 \\
1400 + 0.20x + 0.05y &= 0.10(400)
\end{align*}
\]

(c) \[
\begin{align*}
x &= 0.20(1400) \\
y &= 0.05(400)
\end{align*}
\]

(d) \[
\begin{align*}
0.05x + 0.20y &= 0.10(1400) \\
20x + 0.05y &= 0.15(1800)
\end{align*}
\]

(e) \[
\begin{align*}
x + y &= 400 \\
0.20x + 0.05y &= 0.15(1800)
\end{align*}
\]

10. A concrete walk of uniform width is to be built around a giant circular pool. The radius of the pool is 14 meters, and enough concrete is available to cover 53.64\(\pi\) square meters (approximately). If all the concrete is to be used, how wide should the walk be (approximately)? Choose the closest answer.

Possibilities:

(a) 6.68 meters wide

(b) 3.83 meters wide

(c) 39.6 meters wide

(d) 14 meters wide

(e) 1.8 meters wide

\[
x = \frac{-28 \pm \sqrt{28^2 - 4(4)(53.64)}}{2(4)}
\]

\[
x = \frac{-28 \pm \sqrt{998.56}}{2}
\]

\[
x = 1.8 \text{ meters, } x = -29.8 \text{ meters}
\]
11. Find the equilibrium price. In the supply and demand equations, \( p \) is price (in dollars) and \( x \) is quantity (in thousands). Please round your answer to the nearest hundredth (the nearest cent).

\[
\begin{align*}
\text{Supply: } p &= 6x - 3 \\
\text{Demand: } p &= -9x + 5
\end{align*}
\]

\( \text{Supply } p = \text{Demand } p \)

\[
6x - 3 = -9x + 5 \\
15x = 8 \\
x = \frac{8}{15}
\]

\( \text{ equilibrium price occurs when quantity, } x = \frac{8}{15} \text{ (in thousands).} \)

\[p = -9\left(\frac{8}{15}\right) + 5 = \frac{-72}{15} + \frac{75}{15} = \frac{3}{15} \implies .2 = .2\]

12. A radiator contains 6 quarts of fluid, 25% of which is antifreeze. How much fluid should be drained and replaced with pure (100%) antifreeze so that the new mixture is 55% antifreeze?

Possibilities:

(a) 7.2 quarts drained and replaced
(b) 3.3 quarts drained and replaced
(c) 2.4 quarts drained and replaced
(d) 1.5 quarts drained and replaced
(e) 6 quarts drained and replaced

\( \text{define variable } x = \text{amount to be drained} \)

\( y = \text{amount to be kept} \)

\[
\begin{align*}
x + y &= 6 \\
1.00(x) + .25(y) &= .55(6)
\end{align*}
\]

\[
(x + y = 6)(.25) \Rightarrow -.25x -.25y = -.25(6) \\
+.100x + .25y = .55(6) \\
\]

\[
\begin{align*}
.75x &= 1.8 \\
x &= \frac{1.8}{.75} \\
x &= 2.4 \text{ qts.}
\end{align*}
\]
13. Solve the inequality and express your answer as simplified inequalities.

**LINEAR inequalities**

*can be solved much like LINEAR equations*

Possibilities:
(a) \( x > -1 \) with the exception of the numbers that multiplying or dividing by negatives reverses the inequality symbol
(b) \( x \leq -2 \)
(c) \( x \leq -1 \)
(d) \( x \leq 1 \)
(e) \( x \geq 1 \)

\[
4x + 8 \leq 9x + 3
\]

\[
-9x + 8 \leq 3
\]

\[
-5x \leq -5
\]

\[
x \geq 1
\]

14. Solve the inequality. Express your answer in interval notation.

**Absolute value inequalities must account for both positive and negative values within absolute value**

Possibilities:
(a) \([0, 13]\)
(b) \([5, 13]\)
(c) \([0, \frac{4}{3}]\)
(d) \([-13, -5]\)
(e) \([\frac{4}{3}, 3]\)

\[
|3 + \frac{1}{3}x| \leq \frac{4}{3}
\]

\[
(3 + \frac{1}{3}x) \leq \frac{4}{3}
\]

\[
3 + \frac{1}{3}x \leq \frac{4}{3}
\]

\[
\frac{1}{3}x \leq \frac{5}{3} - 3
\]

\[
x \leq -5
\]

\[
-(3 + \frac{1}{3}x) \leq \frac{4}{3}
\]

\[
3 + \frac{1}{3}x \geq -\frac{4}{3}
\]

\[
\frac{1}{3}x \geq -\frac{4}{3} - 3
\]

\[
x \geq -\frac{13}{3}
\]

\[
\frac{1}{3}x \geq \left(-\frac{13}{3}\right)
\]

\[
x \geq -13
\]
15. Solve the inequality. Answer in interval notation.

Rational inequalities require examination of sign changes for both numerator and denominator.

Possibilities:

(a) [4, 8]
(b) \((-\infty, 4]\)
(c) \((-\infty, 8]\)
(d) \((-\infty, 4]\ \cup \ [8, \infty)\)
(e) [4, 8]

16. Solve the inequality. Answer by choosing the correct number line.

\[0 \leq (x - 4)(x - 3)^2\]

Non-linear inequalities require some examination of sign changes of factors.

Possibilities:

(a) 
(b) 
(c) 
(d) 
(e) 

* Choose & test values in each region to determine whether expression will be + or – in each region.

* Choose regions satisfying inequality.
17. Solve the inequality. Answer by choosing the correct number line.

\[
0 < \frac{(x-4)(x-8)}{(x-9)(x-3)}
\]

* rational inequalities
* require examination
* of sign change in
* numerator & denominator

Possibilities:

(a) [Number line diagram]

(b) [Number line diagram]

(c) [Number line diagram]

(d) [Number line diagram]

(e) [Number line diagram]

18. Find \( f(3) \) from the graph of \( y = f(x) \).

* \( x = 3 \)

Possibilities:

(a) \( f(3) = 7 \)

(b) \( f(3) = 3 \)

(c) \( f(3) = 2 \)

(d) \( f(3) = -1 \)

(e) \( f(3) = 0 \)
19. Find the indicated value of the function when \( x = \sqrt{6} + 2 \).

\[ f(x) = \sqrt{x+8} - x - 3 \]

\[ f(\sqrt{6} + 2) = \]

Possibilities:

(a) \( \sqrt{\sqrt{6} + 10} - \sqrt{6} - 5 \)
(b) 5
(c) \( \sqrt{10} - 5 \)
(d) \( \sqrt{\sqrt{6} + 10} - \sqrt{6} - 1 \)
(e) \( \sqrt{16} - \sqrt{6} - 5 \)

20. Let \( f(x) = 4x^2 + 8 \). Find \( \frac{f(x+h) - f(x)}{h} \) if \( h \neq 0 \). Simplify your answer.

Possibilities:

(a) 16
(b) \( \frac{h+16}{h} \)
(c) \( \frac{4h^2 + 8}{h} \)
(d) \( 8x + 4h \)
(e) \( 4x + 8h \)

* Simplify

\[ \frac{f(x+h) - f(x)}{h} = \frac{4(x^2 + 2xh + h^2) + 8 - 4x^2 - 8}{h} \]

\[ = \frac{4x^2 + 8xh + 4h^2 + 8 - 4x^2 - 8}{h} \]

\[ = \frac{h(8x + 4h)}{h} \]

\[ \Rightarrow 8x + 4h \]