MA109 — College Algebra  Fall 2015  Name:  KEY  Sec.:  
Exam 3  2015-11-18

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. (a) (b) (c) (d) (e)  11. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)  12. (a) (b) (c) (d) (e)
3. (a) (b) (c) (d) (e)  13. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)  14. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)  15. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)  16. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)  17. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)  18. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)  19. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)  20. (a) (b) (c) (d) (e)

For grading use:

<table>
<thead>
<tr>
<th>Number Correct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(out of 20 problems)</td>
<td>(out of 100 points)</td>
</tr>
</tbody>
</table>
Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the indicated value of the function when \( x = \sqrt{6} + 2 \).

   \[ f(x) = \sqrt{x + 8} - x - 3 \]
   \[ f(\sqrt{6} + 2) = \]

   \[ f(\sqrt{6} + 2) = \sqrt{(\sqrt{6} + 2) + 8} - (\sqrt{6} + 2) - 3 \]
   \[ = \sqrt{16} - \sqrt{6} - 2 - 3 \]
   \[ = -\sqrt{6} - 5 \]

   Possibilities:
   (a) \( \sqrt{6} + 10 - \sqrt{6} - 5 \)
   (b) 5
   (c) \( \sqrt{10} - 5 \)
   (d) \( \sqrt{6} + 10 - \sqrt{6} - 1 \)
   (e) \( \sqrt{16} - \sqrt{6} - 5 \)

2. Find \( f(4) \) if \( f(x) = \)

   \[
   \begin{cases} 
   8 & \text{if } x \leq 1 \\
   2x + 6 & \text{if } 1 < x \leq 3 \\
   3x + 3 & \text{if } 3 < x \leq 5 \\
   18 & \text{if } x > 5 
   \end{cases}
   \]

   Possibilities:
   (a) 18
   (b) 8
   (c) 12
   (d) 15
   (e) 14

   \[ f(4) = 3(4) + 3 \]
   \[ = 12 + 3 \]
   \[ = 15 \]
3. Find the domain of \( \sqrt[3]{\frac{x-3}{7}} \)

Possibilities:
(a) \((-\infty, 3) \cup (3, \infty)\)
(b) \((3, \infty)\)
(c) \((-\infty, \infty)\)
(d) \([\frac{3}{7}, \infty)\)
(e) \([3, \infty)\)

\* Domain restriction: red flag, at this point in semester to avoid non-real values \(\rightarrow\) variables under even roots to avoid dividing \(\rightarrow\) variables in denominator by zero

* Root is odd, so no restriction.

* Denominator has no variables, so no restriction.

* No restrictions, so all real numbers or \((-\infty, \infty)\)

4. Find the domain of \( \frac{3}{\sqrt{x-7}} \)

Possibilities:
(a) \((-\infty, 7) \cup (7, \infty)\)
(b) \((7, \infty)\)
(c) \([\frac{3}{7}, \infty)\)
(d) \([7, \infty)\)
(e) \((-\infty, \infty)\)

* Is there an even root? Avoiding negative values under root

* Is there a variable in denominator? Avoiding zero in denominator

\(x-7 > 0\)
\(x > 7\)

NOTE: cannot equal zero in this case because expression is in the denominator of fraction!
5. Which situation below is most reasonable depicted in this graph:

Possibilities:

(a) $y$ is the how many days are left in the semester after $x$ weeks of school, if $x = 0$ is the first week of class.
(b) $y$ is the distance from home at time $x$ as you run to the end of the block and back at a steady pace.
(c) $y$ is the temperature of left-over food at time $x$ if the food is placed in the refrigerator at time $x = 0$.
(d) $y$ is the number of bacteria at time $x$ if the bacteria experience a steady rate of exponential growth.
(e) $y$ is the temperature of an oven at time $x$ if it switched on at time $x = 0$ and left on.

6. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table. Find the average speed of the car from 10 to 15 seconds.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (feet)</td>
<td>0</td>
<td>50</td>
<td>200</td>
<td>450</td>
<td>800</td>
<td>1250</td>
<td>1800</td>
</tr>
</tbody>
</table>

Possibilities:

(a) 20 feet per second
(b) 50 feet per second
(c) 80 feet per second
(d) 60 feet per second
(e) 30 feet per second

**Average speed is**

$$\text{Avg. Speed} = \frac{450 - 200}{15 - 10} = \frac{250}{5} = 50 \text{ feet per second}$$
7. Simplify the formula for the average rate of change of \( f(x) = (x - 3)^2 + 7 \) from \( x = 3 \) to \( x = 3 + h \).

Possibilities:

(a) \( 3 + 2h \)
(b) \( 2h \)
(c) \( 6 + h \)
(d) \( 1 \)
(e) \( h \)

8. Find the domain of \( \left( \frac{f}{g} \right)(x) \) if \( f(x) = 3x^2 + 7x + 8 \) and \( g(x) = 2x - 9 \).

Possibilities:

(a) \( \left[ \frac{9}{2}, \infty \right) \)
(b) \( \left[ \frac{-7 + \sqrt{7^2 - 4(3)(8)}}{6}, \infty \right) \)
(c) \( (-\infty, \frac{9}{2}) \)
(d) \( (-\infty, \infty) \)
(e) \( (-\infty, \frac{9}{2}) \cup \left( \frac{9}{2}, \infty \right) \)

**Domain for \( \left( \frac{f}{g} \right)(x) \) \( \Rightarrow \) \( x \) in domain of \( f \)

\[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2 + 7x + 8}{2x - 9} \]

\( g(x) \neq 0 \)  
\( 2x - 9 \neq 0 \)
\( 2x \neq \frac{9}{2} \)

\( x \) in domain of \( f \)  
\( x \) in domain of \( g \)  
\( x \) in domain of \( \frac{f}{g} \)
9. Find \((f - g)(6)\) where \(f(x) = 4x^2 - 8x - 9\) and \(g(x) = 3x - 2\)

**Possibilities:**

(a) 103
(b) 71
(c) 259
(d) 67
(e) 887

*Subtracting functions is like subtracting the output \(\Rightarrow (f-g)(x) = f(x) - g(x)\)*

\[
(f - g)(6) = f(6) - g(6) \leftarrow \text{plug 6 in for } x
\]

\[
= \left[ 4(6)^2 - 8(6) - 9 \right] - \left[ 3(6) - 2 \right]
\]

\[
= \left[ 4(36) - 48 - 9 \right] - \left[ 18 - 2 \right]
\]

\[
= [87] - [16]
\]

\[
= 71
\]

10. Simplify the formula for \((f \circ g)(x)\) if \(f(x) = 1 - x\) and \(g(x) = \frac{x-1}{x}\)

**Hint:** try plugging in \(x = 9\)

**Possibilities:**

(a) \(\frac{x}{x-1}\)
(b) 9x
(c) \(\frac{1}{x}\)
(d) x
(e) \(\frac{9}{x}\)

*Composition of functions:

\((f \circ g)(x) = f \left( g(x) \right)\)

\[
(f \circ g)(x) = f \left( \frac{x-1}{x} \right)
\]

\[
= 1 - \frac{x-1}{x}
\]

\[
= \frac{x}{x} - \frac{x-1}{x}
\]

\[
= \frac{x - (x-1)}{x}
\]

\[
= \frac{x - x + 1}{x}
\]

\[
= \frac{1}{x}
\]

Using the hint:

\[(f \circ g)(9) = f \left( g(9) \right) = f \left( \frac{2}{9} \right) = 1 - \frac{2}{9} = \frac{7}{9}\]
11. Suppose that the graph of \( y = f(x) \) contains the point \((4, 8)\). Find a point that must be on the graph of \( y = g(x) \) for \( g(x) = 9 + f(3x + 2) \).

**Possibilities:**

(a) \( \left( \frac{2}{3}, -1 \right) \)

(b) \((14, 17)\)

(c) \(\left( -\frac{2}{3}, -1 \right)\)

(d) \(\left( \frac{2}{3}, 17 \right)\)

(e) \(\left( 14, -1 \right)\)

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12. Which sequence of transformations will transform the graph of the function \( f \) into the graph of the function \( g \)?

\[
f(x) = \sqrt{x} + 4 \quad \text{and} \quad g(x) = \sqrt{x - 3} + 6
\]

**Possibilities:**

(a) shift right by 3 then shift down by 2

(b) shift left by 3 then shift up by 2

(c) shift left by 2 then shift down by 3

(d) shift right by 3 then shift up by 2

(e) shift left by 3 then shift down by 2

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The page number is 7.
13. Use algebra to find the inverse of the given one-to-one function.

\[ f(x) = (x^5 + 9)^4 \]

\*to find inverse function \[ f(x) = (x^5 + 9)^4 \]

\[ y = (x^5 + 9)^4 \]

\[ \sqrt[4]{y} = x^5 + 9 \]

\[ x = \sqrt[4]{y} - 9 \]

\[ f^{-1}(x) = \sqrt[4]{y} - 9 \]

**NOTE:** Technically, should be \( \pm \sqrt[4]{x} \), but that is NOT 1-1. So, we'll assume just positive root for this solution.

14. Use algebra to find the inverse of the given one-to-one function. \[ f(x) = \frac{4x}{8x + 9} \]

Possibilities:

(a) \[ f^{-1}(x) = \frac{1}{2}x + 9 \]

(b) \[ f^{-1}(x) = \frac{4x}{8x - 9} \]

(c) \[ f^{-1}(x) = \frac{9x}{4 - 8x} \]

(d) \[ f^{-1}(x) = \frac{8x + 9}{4x} \]

(e) \[ f^{-1}(x) = \frac{9x}{4x + 8} \]
15. Write the given expression without using radicals.

Possibilities:
(a) $x^5 - x^7$
(b) $x^2$
(c) $x^{-2}$
(d) $x^{7/5}$
(e) $x^{5/7}$

√

16. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week: 1 2 3 4 5 6
Frogs: 45 135 405 1215 3645 10935

Which exponential growth model most closely matches the observations, if \( t \) is the week number?

Possibilities:
(a) 15 \((9^{t/7})\)
(b) 3 \((45^t)\)
(c) 15 \((3^t)\)
(d) 3 \((45^{t/7})\)
(e) 45 \((9^t)\)
17. Determine how much money (to the nearest cent) will be in a savings account if the initial deposit was $2000 and the interest rate is 3.250% compounded continuously for 7 years.

Possibilities:
(a) $2510.82
(b) $2510.85
(c) $2510.88
(d) $2510.91
(e) $2510.94

\[ P(t) = P_0 e^{rt} \]

\[ P(7) = 2000e^{0.0325 \times 7} \]

\[ P(7) = 2000e^{0.2275} \]

\[ P(7) = 2510.91 \]

18. Translate the given exponential statement into an equivalent logarithmic statement.

\[ 4^x = 8 \]

*Definition of log function: \[ a^x = y \] if and only if \[ \log_a(y) = x \]

Possibilities:
(a) \( \log_4(8) = x \)
(b) \( \log_8(4) = x \)
(c) \( \log_8(x) = 4 \)
(d) \( \log_x(4) = 8 \)
(e) \( \log_4(x) = 8 \)
19. Write the domain of the function \( h(x) = \log(x - 3) \) in interval notation.

**Possibilities:**
(a) \(( -\infty, 3 ) \cup (3, \infty)\)
(b) \(( -\infty, -3)\)
(c) \(( -\infty, 3]\)
(d) \(( -\infty, \infty)\)
(e) \((3, \infty)\)

\[ x - 3 > 0 \]
\[ x > 3 \]
\[ (3, \infty) \]

**Log functions require input values that are strictly positive!**

20. Write the given expression as a single logarithm.

\[ 4 \log(x) + \log(8y) - \log(9z) \]

**Possibilities:**
(a) \( \log \left( \frac{x^4(8y)}{9z} \right) \)
(b) \( \log \left( x^4y^8z^9 \right) \)
(c) \( \log \left( \frac{x^4y^8}{z^9} \right) \)
(d) \( \log (4x + 8y - 9z) \)
(e) \( \log \left( 4x(8 + y) - 9z \right) \)

**Rewrite sum as product**
**Rewrite difference as quotient**
**Log property:**
\[ \log_a(xy) = \log_a(x) + \log_a(y) \]
**Log property:**
\[ \log_a\left( \frac{x}{y} \right) = \log_a(x) - \log_a(y) \]
**Formula Sheet:**

**Compound Interest:** If a principal $P_0$ is invested at an interest rate $r$ for a period of $t$ years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded $n$ times per year)}$$

$$P(t) = P_0 \ e^{rt} \quad \text{(if compounded continuously)}.$$

**Change of Base Formula:** Let $a$ and $b$ be two positive numbers with $a,b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$