

Table of contents

Inequalities

Equivalent Inequalities

Solving a Linear or Nonlinear Inequality

Absolute Value Inequalities

Concepts

- Equivalent Inequalities
- Linear and Nonlinear Inequalities
- Absolute Value Inequalities

(Section 4.6 and 1.1)

Equivalent Inequalities

Definition 1

Two inequalities are **equivalent** if they have the same solution set.

Equivalent Inequalities

Operations that Produce Equivalent Inequalities.

- Add or Subtract the same value on both sides of the inequality.
- Multiply or Divide by the same **positive value** on both sides of the inequality.
- Multiply or Divide by the same **negative value** on both sides of the inequality **AND change the direction of the inequality.**

Equivalent Inequalities

Example 2

For each pair of inequalities, determine if the two inequalities are equivalent.

● $x^2 + 2x \leq 5$ and $x^2 + 2x - 5 \leq 0$.

● $6 - x < 5$ and $x > 1$

● $\frac{x}{x+2} > 3$ and $\frac{x}{x+2} - 3 > 0$

● $\frac{x}{x+2} > 3$ and $x > 3(x+2)$

Equivalent Inequalities

You should **NEVER** multiply both sides of an inequality by an expression involving x that you don't know the sign of. Why?

Solving a Linear or Nonlinear Inequality

Example 3 (Linear Inequality)

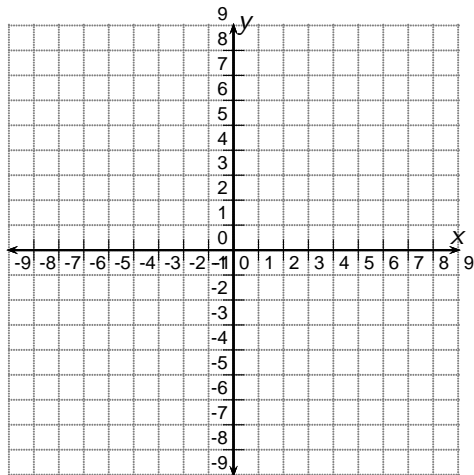
Solve the inequality $5x + 3 \leq 6 - 7x$. Write the solution set in interval notation.

Solving a Linear or Nonlinear Inequality

There are many ways to solve this inequality algebraically. We will begin by using addition and subtraction to move all the nonzero quantities to one side. This is not necessary for this inequality, but it will help us to understand the process needed for solving more complicated inequalities.

Solving a Linear or Nonlinear Inequality

Graph the equations $y = 5x + 3$ and $y = 6 - 7x$. How can you approximate the solutions of an inequality graphically?



Solving a Linear or Nonlinear Inequality

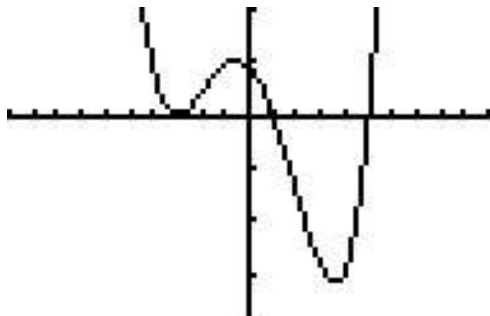
Thinking graphically can help us understand the algebraic procedure for solving Nonlinear Inequalities. Given an expression, such as $(x + 3)^2(x - 1)(x - 5)$, the expression is positive when the graph of $y = (x + 3)^2(x - 1)(x - 5)$ is _____ . The expression is negative when the graph of $y = (x + 3)^2(x - 1)(x - 5)$ is _____ .

Solving a Linear or Nonlinear Inequality

Example 4 (Polynomial Inequality)

The graph of $y = (x + 3)^2(x - 1)(x - 5)$ is shown below. The viewing window is $[-10, 10] \times [-200, 100]$. Use the graph to help you approximate the solutions of the inequality.

$$(x + 3)^2(x - 1)(x - 5) > 0$$



Solving a Linear or Nonlinear Inequality

Algebraic Procedure for Solving Linear and Nonlinear Inequalities

1. Use addition and subtraction to move all nonzero quantities to one side. If fractional expressions are involved, simplify so the nonzero side is a single fractional expression.
2. Find the zeros of the expression AND the zeros of all denominators. (If you can factor the expression, this can help. Finding a zero means find when the expression equals zero.)
3. Make a sign chart to determine if the values between the zeros from step 2 lead to positive or negative values of the polynomial.
4. Answer the question.

Solving a Linear or Nonlinear Inequality

Example 5

Use the algebraic approach to solve $(x + 3)^2(x - 1)(x - 5) > 0$. Be sure to write your answer in interval notation.

Solving a Linear or Nonlinear Inequality

Example 6 (Quadratic Inequality)

Solve the inequality below. Be sure to write your answer in interval notation.

$$x^2 + 2x > 8$$

Solving a Linear or Nonlinear Inequality

Example 7 (Rational Inequality)

Solve the inequality below. Be sure to write your answer in interval notation.

$$\frac{2}{x+3} \leq \frac{4}{x-1}$$

Solving a Linear or Nonlinear Inequality

Example 8 (Inequality Application)

A computer store has determined that the cost C of ordering and storing x laser printers is given by $C = 2x + \frac{300,000}{x}$. If the delivery truck can bring at most 450 printers per order, how many printers should be ordered at a time to keep the cost below \$1600?

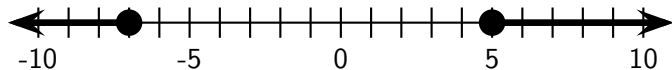
Absolute Value Inequalities

Example 9 (Another Distance Example)

Solve $|x + 3| > 5$ geometrically. Be sure to write your answer in interval notation.

Absolute Value Inequalities

Example 10 (Another Distance Example)



- (a) Write a distance sentence that corresponds to this number line.
- (b) Write an absolute value equation or inequality that corresponds to this number line.

Absolute Value Inequalities

Example 11 (The Algebraic Approach to Absolute Values)

Solve each inequality algebraically.

(a) $|2x + 3| \geq 7$

Absolute Value Inequalities

$$(b) |x + 2| + 1 < 3$$