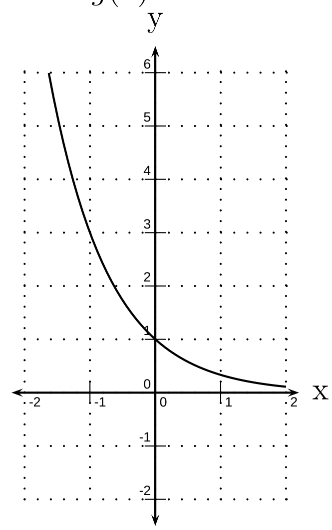
11 Exponential and Logarithmic Functions Worksheet

Concepts:

- Rules of Exponents
- Exponential Functions
  - Power Functions vs. Exponential Functions
  - The Definition of an Exponential Function
  - Graphing Exponential Functions
  - Exponential Growth and Exponential Decay
- Compound Interest
- Logarithms
  - Logarithms with Base a
    - Definition
    - Exponential Notation vs. Logarithmic Notation
    - Evaluating Logarithms
    - Graphs of Logarithms
    - Domain of Logarithms
  - Properties of Logarithms
    - Simplifying Logarithmic Expressions
    - Using the Change of Base Formula to Find Approximate Values of Logarithms
- Solving Exponential and Logarithmic Equations

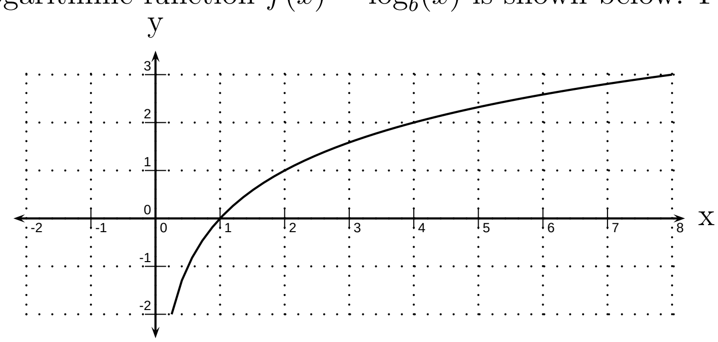
(Chapter 5)

1. The graph of an exponential function $g(x) = a^x$ is shown below. Find a.



$a = \dfrac{1}{3}$

2. The graph of a logarithmic function $f(x) = \log_b(x)$ is shown below. Find b.



$b = 2$

3. Layney invests \$1000 at an interest rate of 5% per year compounded semi-annually. How much money will be in her account after 18 years? **Approximately** \$2432.54**.**

4. Layney invests \$1000 at an interest rate of 5% per year compounded continuously. How much money will be in her account after 18 years? **Approximately** \$2459.60**.**

5. Find the average rate of change of $f(x) = 2^x$ from $x = 3$ to $x = 4$. Draw a graph that illustrates the meaning of your answer. **8**

6. Find the exact value of the following logarithms. Do NOT use your calculator.

   (a) $\log_3(27) = \mathbf{3}$ (b) $\log(\sqrt[3]{100}) = \dfrac{2}{3}$

   (c) $\log_5\left(\dfrac{1}{25}\right) = -2$ (d) $\ln(\sqrt[5]{e^3}) = \dfrac{3}{5}$ (e) $e^{2\ln(x)} = x^2$

7. Write each expression in terms of $\log(x)$, $\log(y)$, and $\log(z)$ if possible. If it is not possible, explain why.

   (a) $\log\left(\frac{x^3y^7}{\sqrt{z}}\right)$
   $3\log(x)+7\log(y)-\frac{1}{2}\log(z)$

   (b) $\log\left(\frac{x^2+y^2}{z}\right)$
   $\log(x^2+y^2)-\log(z)$

   (c) $\log\left(x^5\sqrt[3]{yz}\right)$
   $5\log(x)+\frac{1}{3}\log(y)+\frac{1}{3}\log(z)$

8. Convert each exponential statement to an equivalent logarithmic statement.

   (a) $2^x=16$
   $\log_2(16)=x$

   (b) $3^4=y$
   $\log_3(y)=4$

9. Convert each logarithmic statement to an equivalent exponential statement.

   (a) $\log_2(32)=5$
   $2^5=32$

   (b) $\ln(x)=3$
   $e^3=x$

   (c) $\log(9)=x$
   $10^x=9$

10. Use your calculator to find approximate values for the following.

   (a) $\ln(7)\approx 1.94591$

   (b) $\log(53)\approx 1.72428$

   (c) $\log_5(6)\approx 1.11328$

   (d) $\log_2(21)\approx 4.39232$

11. Find all real solutions or state that there are none. Your answers should be exact.

   (a) $4e^{x-8}=2$
   $x=\ln(0.5)+8$

   (b) $4^{x+1}=16$
   $x=1$

(c) $\log_8(x-5)+\log_8(x+2)=1$
$x=6$

(d) $2\log(2x)=4$
$x=50$

12. When a living organism dies, its carbon-14 decays. The half life of carbon-14 is 5730 years. If the skeleton of a human is discovered and has 20% of its original carbon-14 remaining, how long ago did the human die? **Approximately 13,305 years ago**

13. Joni invests \$5000 at an interest rate of 5% per year compounded continuously. How much time will it take for the value of the investment to quadruple? **Approximately 27.72589 years**

14. Joni invests \$5000 at an interest rate of 5% per year compounded monthly. How much time will it take for the value of the investment to quadruple? **Approximately 27.78361 years**

15. Let $f(x)=\ln(3x+7)$. Find $f^{-1}(x)$.
$f^{-1}(x)=\dfrac{e^x-7}{3}$

16. Let $f(x)=2^{x+3}-1$. Find $f^{-1}(x)$.
$f^{-1}(x)=\log_2(x+1)-3$

17. Find the domain of $f(x)=\ln(2-3x)$
$\left(-\infty,\dfrac{2}{3}\right)$

18. Find the domain of $g(x)=\dfrac{x}{\ln(5x+4)}$
$\left(-\dfrac{4}{5},-\dfrac{3}{5}\right)\cup\left(-\dfrac{3}{5},\infty\right)$

19. Find the domain of $h(x)=\ln(x^2-2x-15)$
$(-\infty,-3)\cup(5,\infty)$

20. **The Paper-Folding Problem**

Take a piece of paper and fold it in half. Now fold it in half again. And again. How many times can you fold it in half? Notice how quickly the thickness of the folded paper increases.

(a) Suppose you want to write a function that models the thickness t of the paper after you fold it n times. What additional information do you need to write this function.

(b) Assume that the paper you are folding is $8\frac{1}{2}$ inches by 11 inches and the paper is 0.1 mm thick.

i. Write a function that models the thickness t of the paper after you fold it n times.

ii. What are the units for t?

iii. What are the units for n?

iv. What is the domain of $t(n)$?

(c) How thick is the folded paper after 5 folds?

(d) How thick is the folded paper after 10 folds?

(e) How many folds are needed for the paper to be at least as thick as a notebook? (The thickness of a notebook is about 12.8 mm.)

(f) How many folds are needed for the paper to be at least as thick as an average person is tall? (The height of an average person is 1.6m.)

(g) How many folds are needed for the paper to be at least as thick as the Sears tower is tall? (The height of the Sears Tower is about 440m.)

(h) How many folds are needed for the paper to reach to the Sun? (The sun is about 95 million miles away. There are about 1.609344 km in 1 mile.)