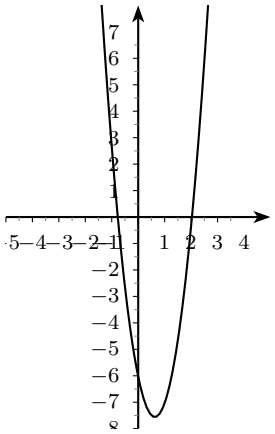
7.1 Polynomials Practice Problems

1. Let $f(x) = x^2$ and $g(x) = 4x^2 - 5x - 6$.

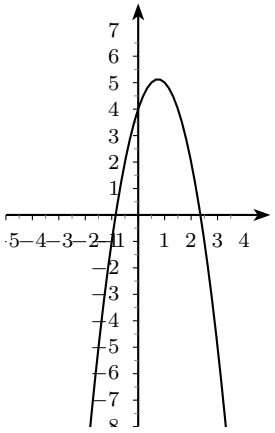
   - Describe the transformations that could be applied to the graph of f to obtain the graph of g. $\mathbf{g(x) = 4f(x - \frac{5}{8}) - \frac{121}{16}}$
     - **Shift right** $\frac{5}{8}$
     - **Scale vertically by a factor of 4**
     - **Shift down** $\frac{121}{16}$
   - Sketch the graph of g.



   - What is the vertex of the graph of g? $\left(\mathbf{\frac{5}{8}, -\frac{121}{16}}\right)$
   - Does the graph of g have an absolute minimum or an absolute maximum? What is it? **Absolute minimum of** $\mathbf{-\frac{121}{16}}$

2. Let $f(x) = x^2$ and $g(x) = -2x^2 + 3x + 4$.

   - Describe the transformations that could be applied to the graph of f to obtain the graph of g. $\mathbf{g(x) = -2f(x - \frac{3}{4}) + \frac{41}{8}}$
   - Sketch the graph of g.



- What is the vertex of the graph of g? $\left(\mathbf{\frac{3}{4}}, \mathbf{\frac{41}{8}}\right)$
- Does the graph of g have an absolute minimum or an absolute maximum? What is it? **Absolute maximum of $\mathbf{\frac{41}{8}}$**

3. Find the quadratic function $f(x) = ax^2 + bx + c$ whose vertex is $(2, 3)$ and goes through the point $(-6, 4)$. $\mathbf{\frac{1}{64}(x-2)^2 + 3 = \frac{1}{64}x^2 - \frac{1}{16}x + \frac{49}{16}}$

4. (Exercise 55 from Section 4.1 of your textbook) A field bounded on one side by a river is to be fenced on three sides so as to form a rectangular enclosure. If 200 feet of fencing is to be used, what dimensions will yield an enclosure of the largest possible area? **50 feet (side perpendicular to the river by 100 feet (side parallel to the river)**

5. (Exercise 56 from Section 4.1 of your textbook) A rectangular box (with top) has a square base. The sum of the lengths of its 12 edges is 8 feet. What dimensions should the box have so that its surface area is as large as possible? $\mathbf{\frac{2}{3}}$ feet by $\mathbf{\frac{2}{3}}$ feet by $\mathbf{\frac{2}{3}}$ feet

6. (Exercise 61 from Section 4.1 of your textbook) A potter can sell 120 bowls per week at \$5 per bowl. For each 50 cent decrease in price, 20 more bowls are sold. What price should be charged to maximize sales income. **\$2.50**

7. Describe the end behavior of each polynomial. Use correct mathematical symbols.

   (a) $P(x) = 2x^5 - 3x^2 + 76$

   $y \to \infty$ as $x \to \infty$ and

   $y \to -\infty$ as $x \to -\infty$.

   (b) $R(x) = (2x+3)^4(50-x)^{100}$ **Leading term:** $(2x)^4(-x)^{100}$

   $y \to \infty$ as $x \to \infty$ and

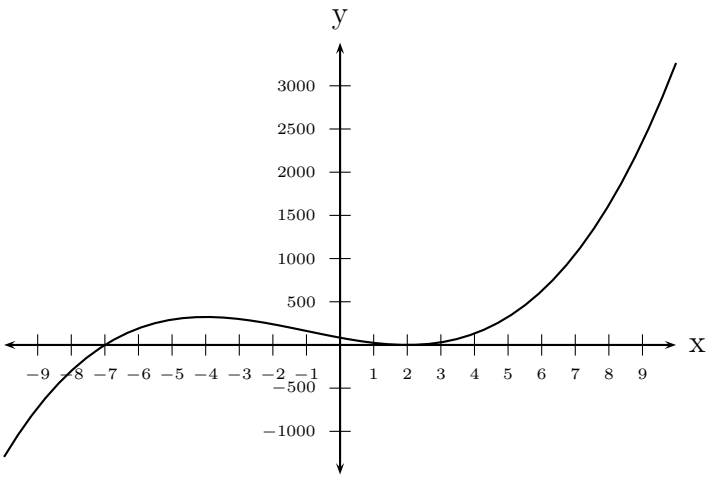
   $y \to \infty$ as $x \to -\infty$.

   (c) $S(x) = (1-2x)^{11}(x+5)^4$ **Leading term:** $(-2x)^{11}(x)^4$

   $y \to -\infty$ as $x \to \infty$ and

   $y \to \infty$ as $x \to -\infty$.

8. The graph of a polynomial $P(x)$ is shown below.



(a) Is the degree of the polynomial even or **odd**?

(b) Is the leading coefficient **positive** or negative?

(c) What can you say about the factors of this polynomial? $(\mathbf{x+7})$ **is a factor and has an odd exponent,** $(\mathbf{x-2})$ **is a factor and has an even exponent**

(d) Can you find a formula for the polynomial if you know that the degree of the polynomial is less than or equal to 4 and that $P(1) = 24$? $\mathbf{P(x) = \frac{6}{7}(x+7)(x-2)^2}$

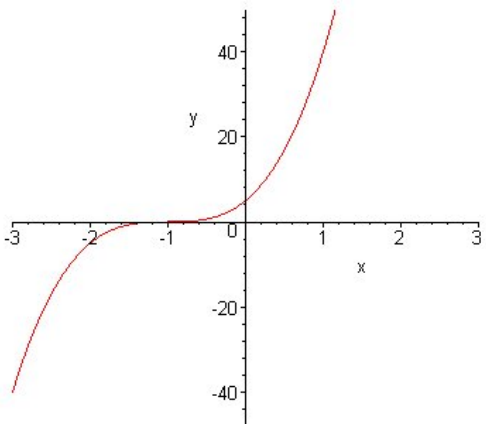
9. The graph shown below is NOT the graph of $y = h(x) = 5(x+1)^4$. Which of the following are clues that this is NOT the graph of h?

(a) **The graph crosses the** x**-axis at** $(-1, 0)$**, but it should not cross the** x**-axis at this point.**

(b) **The graph displays the wrong end behavior.**

(c) The graph has the wrong x-intercepts.

(d) **The graph does not have the right number of local extreme points to be the graph of a polynomial of degree** 4**.**



10. Find the quotient and the remainder for each of the following division problems.

    (a) $\dfrac{x^3 - 2x^2 + 7x + 1}{2x + 3}$

    (b) $\dfrac{x^4 + 3x - 1}{x^2 + 2}$ **Quotient: $\mathbf{x^2 - 2}$, Remainder:$\mathbf{3x + 3}$**

    (c) $\dfrac{2x + 3}{x^3 - 2x^2 + 7x + 1}$ **Quotient: 0, Remainder:$\mathbf{2x + 3}$**

11. Find the remainder of the following division problems.

    (a) $\dfrac{x^{99} + 7}{x - 1}$ **Remainder:8**

    (b) $\dfrac{x^{100000} + x^{99999} + 4}{x + 1}$ **Remainder:4**

    (c) $\dfrac{x^{10} - 3}{x + 2}$ **Remainder:1021**

12. Find all the real roots of each polynomial.

    (a) $P(x) = 2x^4 - 17x^3 + 31x^2 - 85x + 105$ **Real roots: 7, $\frac{3}{2}$**

    (b) $Q(x) = 9x^4 - 30x^3 + 10x^2 + 16x - 9$ **Real roots:** $\approx -7.480045, 2.73950$ **using graphing calculator**