Worksheet 11 – Logarithm Rules (§5.4)

1. Fill in the blank.

(a) $\log_2(8) + \log_2(4) = \log_2(\square)$

(b) $\log_2(64) - \log_2(4) = \log_2(\square)$

(c) $3\log_2(4) = \log_2\left(4^{\square}\right)$

(d) $\log_3(3^{-2}) = \square$

(e) $2^{\log_2(16)} = \square$

(f) $\log_9(81) = \frac{\log_3(\square)}{\log_3(\square)} = \frac{\log_{81}(\square)}{\log_{81}(\square)}$

2. Complete the following rules for any numbers $M, N > 0$.

(a) $\log_b(M) + \log_b(N) = \square$

(b) $\log_b(M) - \log_b(N) = \square$

(c) $r\log_b(M) = \square$

(d) $\log_b(b^x) = \square$

(e) $b^{\log_b(x)} = \square$ if $x > \square$

(f) $\log_b(M) = \square$ for any $c > 0$

3. Determine whether the following statements as true or false.

(a) $\log_3(3^{-7}) = -7$

(b) $3^{\log_3(-7)} = -7$

(c) $\log_4(5) + \log_4(6) = \log_4(11)$

(d) $\log_5(-4) + \log_5(-3) = \log_5(12)$

(e) $\log(6^{-2}) = -2\log(6)$

(f) $\ln(0) = 1$

4. Simplify the following expressions in terms of $\log(3)$.

(a) $\log(3^7)$

(b) $\log(\sqrt{3})$

(c) $\log(81)$

(d) $\log\left(\frac{1}{27}\right)$

5. Simplify the following expressions in terms of $\log(2)$ and $\log(3)$.

(a) $\log(6)$

(b) $\log\left(\frac{16}{3}\right)$

(c) $\log\left(\frac{9}{4}\right)$

(d) $\log\left(\frac{1}{162}\right)$

6. Simplify the following expressions (assume all variables are positive).

(a) $e^{\ln(6x)}$

(b) $\log_7(7^{2x+1})$

(c) $w^{\log_w(yz)}$

(d) $\log(1000^t)$

7. Write the following expressions using sums/differences of multiples of logarithms.

(a) $\ln\left(\frac{3x^2}{yz}\right)$

(b) $\log_3\left(\frac{(x-1)^2}{z^{3/2}}\right)$

(c) $\log\left(\frac{100x\sqrt{y}}{\sqrt[3]{10}}\right)$

(d) $\log_6\left(\frac{216}{x^3y}\right)^4$

8. Write the following expressions as a single logarithm (assume all variables are positive).

(a) $\log_3(x) + \log_3(6)$

(b) $\ln(3) + \ln(x^2) + \ln(y)$

(c) $\ln(x^6) - \ln(x^2)$

(d) $\ln(x+1) + \ln(3) - 3\ln(y)$

(e) $\frac{1}{2}\log(z+1) - \frac{1}{3}\log(w) + 4\log(x)$

(f) $\log_3(90) - 2$