Worksheet 16 - The Six Trigonometric Functions (§6.6 and §7.1)

In Exercises 1 - 20, **compute** the exact value or state that it is undefined.

1. $\tan\left(\frac{\pi}{4}\right)$
2. $\sec\left(\frac{\pi}{6}\right)$
3. $\csc\left(\frac{5\pi}{6}\right)$
4. $\cot\left(\frac{4\pi}{3}\right)$
5. $\tan\left(-\frac{11\pi}{6}\right)$
6. $\sec\left(-\frac{3\pi}{2}\right)$
7. $\csc\left(-\frac{\pi}{3}\right)$
8. $\cot\left(\frac{13\pi}{2}\right)$
9. $\tan(117\pi)$
10. $\sec\left(-\frac{5\pi}{3}\right)$
11. $\csc(3\pi)$
12. $\cot(-5\pi)$
13. $\tan\left(\frac{31\pi}{2}\right)$
14. $\sec\left(\frac{\pi}{4}\right)$
15. $\csc\left(-\frac{7\pi}{4}\right)$
16. $\cot\left(\frac{7\pi}{6}\right)$
17. $\tan\left(\frac{2\pi}{3}\right)$
18. $\sec(-7\pi)$
19. $\csc\left(\frac{\pi}{2}\right)$
20. $\cot\left(\frac{3\pi}{4}\right)$

In Exercises 21 - 34, use the given the information to **compute** the exact values of the remaining **trigonometric** functions of θ.

21. $\sin(\theta) = \frac{3}{5}$ with θ in Quadrant II
22. $\tan(\theta) = \frac{12}{5}$ with θ in Quadrant III
23. $\csc(\theta) = \frac{25}{24}$ with θ in Quadrant I
24. $\sec(\theta) = 7$ with θ in Quadrant IV
25. $\csc(\theta) = -\frac{10\sqrt{91}}{91}$ with θ in Quadrant III
26. $\cot(\theta) = -23$ with θ in Quadrant II
27. $\tan(\theta) = -2$ with θ in Quadrant IV.
28. $\sec(\theta) = -4$ with θ in Quadrant II.
29. $\cot(\theta) = \sqrt{5}$ with θ in Quadrant III.
30. $\cos(\theta) = \frac{1}{3}$ with θ in Quadrant I.
31. $\cot(\theta) = 2$ with $0 < \theta < \frac{\pi}{2}$.
32. $\csc(\theta) = 5$ with $\frac{\pi}{2} < \theta < \pi$.
33. $\tan(\theta) = \sqrt{10}$ with $\pi < \theta < \frac{3\pi}{2}$.
34. $\sec(\theta) = 2\sqrt{5}$ with $\frac{3\pi}{2} < \theta < 2\pi$.

In Exercises 35 - 42, use your calculator **or computer** to approximate the given value to three decimal places. Make sure your **device** is in the proper angle measurement mode!

35. $\csc(78.95^\circ)$
36. $\tan(-2.01)$
37. $\cot(392.994)$
38. $\sec(207^\circ)$
39. $\csc(5.902)$
40. $\tan(39.672^\circ)$
41. $\cot(3^\circ)$
42. $\sec(0.45)$

In Exercises 43 - 57, **compute** all of the angles which satisfy the equation.

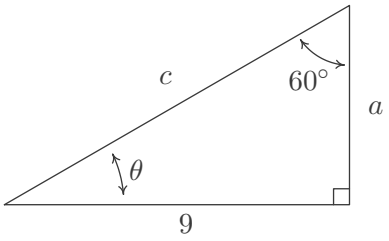
43. $\tan(\theta) = \sqrt{3}$
44. $\sec(\theta) = 2$
45. $\csc(\theta) = -1$
46. $\cot(\theta) = \dfrac{\sqrt{3}}{3}$
47. $\tan(\theta) = 0$
48. $\sec(\theta) = 1$
49. $\csc(\theta) = 2$
50. $\cot(\theta) = 0$
51. $\tan(\theta) = -1$
52. $\sec(\theta) = 0$
53. $\csc(\theta) = -\dfrac{1}{2}$
54. $\sec(\theta) = -1$
55. $\tan(\theta) = -\sqrt{3}$
56. $\csc(\theta) = -2$
57. $\cot(\theta) = -1$

In Exercises 58 - 65, solve the equation for t. Give exact values.

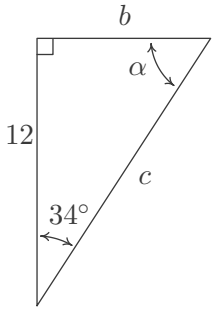
58. $\cot(t) = 1$
59. $\tan(t) = \dfrac{\sqrt{3}}{3}$
60. $\sec(t) = -\dfrac{2\sqrt{3}}{3}$
61. $\csc(t) = 0$
62. $\cot(t) = -\sqrt{3}$
63. $\tan(t) = -\dfrac{\sqrt{3}}{3}$
64. $\sec(t) = \dfrac{2\sqrt{3}}{3}$
65. $\csc(t) = \dfrac{2\sqrt{3}}{3}$

In Exercises 66 - 69, **compute** the requested quantities.

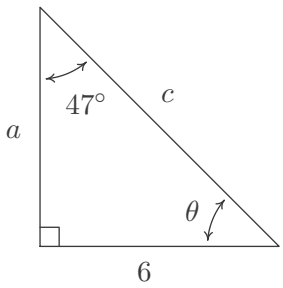
66. **Compute** θ, a, and c.



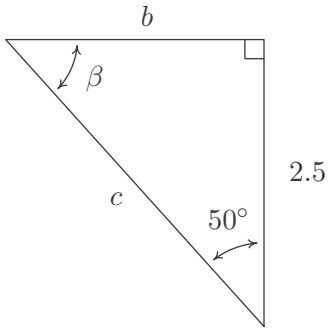
67. **Compute** α, b, and c.



68. **Compute** θ, a, and c.



69. **Compute** β, b, and c.



In Exercises 70 - 75, answer the question. Assume that θ is an angle in a right triangle.

70. If $\theta = 30°$ and the side opposite θ has length 4, how long is the side adjacent to θ?

71. If $\theta = 15°$ and the hypotenuse has length 10, how long is the side opposite θ?

72. If $\theta = 87°$ and the side adjacent to θ has length 2, how long is the side opposite θ?

73. If $\theta = 38.2°$ and the side opposite θ has lengh 14, how long is the hypoteneuse?

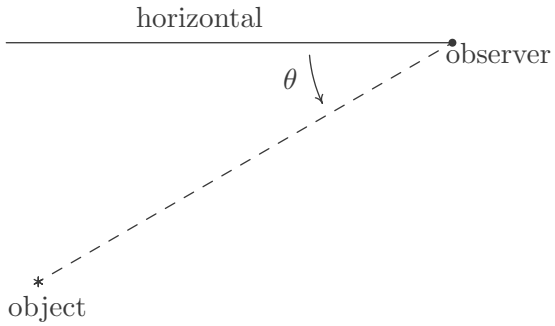
74. If $\theta = 2.05°$ and the hypotenuse has length 3.98, how long is the side adjacent to θ?

75. If $\theta = 42°$ and the side adjacent to θ has length 31, how long is the side opposite θ?

76. A tree standing vertically on level ground casts a 120 foot long shadow. The angle of elevation from the end of the shadow to the top of the tree is 21.4°. **Compute** the height of the tree to the nearest foot.

77. The broadcast tower for radio station WSAZ has two enormous flashing red lights on it: one at the very top and one a few feet below the top. From a point 5000 feet away from the base of the tower on level ground the angle of elevation to the top light is 7.970° and to the second light is 7.125°. **Compute** the distance between the lights to the nearest foot.

78. The angle of depression of an object refers to the angle whose initial side is a horizontal line above the object and whose terminal side is the line-of-sight to the object below the horizontal. This is represented schematically below.



The angle of depression from the horizontal to the object is θ

(a) If the horizontal is above and parallel to level ground, then **explain why** the angle of depression (from observer to object) and the angle of inclination (from object to observer) will be congruent.

(b) From a firetower 200 feet above level ground in the Sasquatch National Forest, a ranger spots a fire off in the distance. The angle of depression to the fire is 2.5°. How far away from the base of the tower is the fire?

(c) The ranger in part 78b sees a Sasquatch running directly from the fire towards the firetower. The ranger takes two sightings. At the first sighting, the angle of depression from the tower to the Sasquatch is 6°. The second sighting, taken just 10 seconds later, gives the the angle of depression as 6.5°. How far did the Saquatch travel in those 10 seconds? Round your answer to the nearest foot. How fast is it running in miles per hour? Round your answer to the nearest mile per hour. If the Sasquatch keeps up this pace, how long will it take for the Sasquatch to reach the firetower from his location at the second sighting? Round your answer to the nearest minute.

79. When I stand 30 feet away from a tree at home, the angle of elevation to the top of the tree is 50° and the angle of depression to the base of the tree is 10°. What is the height of the tree? Round your answer to the nearest foot.

80. From the observation deck of the lighthouse at Sasquatch Point 50 feet above the surface of Lake Ippizuti, a lifeguard spots a boat out on the lake sailing directly toward the lighthouse. The first sighting had an angle of depression of 8.2° and the second sighting had an angle of depression of 25.9°. How far had the boat traveled between the sightings?

81. A guy wire 1000 feet long is attached to the top of a tower. When pulled taut it makes a 43° angle with the ground. How tall is the tower? How far away from the base of the tower does the wire hit the ground?

In Exercises 82 - 128, verify the identity. Assume that all quantities are defined.

82. $\cos(\theta)\sec(\theta) = 1$

83. $\tan(\theta)\cos(\theta) = \sin(\theta)$

84. $\sin(\theta)\csc(\theta) = 1$

85. $\tan(\theta)\cot(\theta) = 1$

86. $\csc(\theta)\cos(\theta) = \cot(\theta)$

87. $\dfrac{\sin(\theta)}{\cos^2(\theta)} = \sec(\theta)\tan(\theta)$

88. $\dfrac{\cos(\theta)}{\sin^2(\theta)} = \csc(\theta)\cot(\theta)$

89. $\dfrac{1+\sin(\theta)}{\cos(\theta)} = \sec(\theta) + \tan(\theta)$

90. $\dfrac{1-\cos(\theta)}{\sin(\theta)} = \csc(\theta) - \cot(\theta)$

91. $\dfrac{\cos(\theta)}{1-\sin^2(\theta)} = \sec(\theta)$

92. $\dfrac{\sin(\theta)}{1-\cos^2(\theta)} = \csc(\theta)$

93. $\dfrac{\sec(\theta)}{1+\tan^2(\theta)} = \cos(\theta)$

94. $\dfrac{\csc(\theta)}{1+\cot^2(\theta)} = \sin(\theta)$

95. $\dfrac{\tan(\theta)}{\sec^2(\theta)-1} = \cot(\theta)$

96. $\dfrac{\cot(\theta)}{\csc^2(\theta)-1} = \tan(\theta)$

97. $4\cos^2(\theta) + 4\sin^2(\theta) = 4$

98. $9 - \cos^2(\theta) - \sin^2(\theta) = 8$

99. $\tan^3(\theta) = \tan(\theta)\sec^2(\theta) - \tan(\theta)$

100. $\sin^5(\theta) = \left(1 - \cos^2(\theta)\right)^2 \sin(\theta)$

101. $\sec^{10}(\theta) = \left(1 + \tan^2(\theta)\right)^4 \sec^2(\theta)$

102. $\cos^2(\theta)\tan^3(\theta) = \tan(\theta) - \sin(\theta)\cos(\theta)$

103. $\sec^4(\theta) - \sec^2(\theta) = \tan^2(\theta) + \tan^4(\theta)$

104. $\dfrac{\cos(\theta)+1}{\cos(\theta)-1} = \dfrac{1+\sec(\theta)}{1-\sec(\theta)}$

105. $\dfrac{\sin(\theta)+1}{\sin(\theta)-1} = \dfrac{1+\csc(\theta)}{1-\csc(\theta)}$

106. $\dfrac{1-\cot(\theta)}{1+\cot(\theta)} = \dfrac{\tan(\theta)-1}{\tan(\theta)+1}$

107. $\dfrac{1-\tan(\theta)}{1+\tan(\theta)} = \dfrac{\cos(\theta)-\sin(\theta)}{\cos(\theta)+\sin(\theta)}$

108. $\tan(\theta) + \cot(\theta) = \sec(\theta)\csc(\theta)$

109. $\csc(\theta) - \sin(\theta) = \cot(\theta)\cos(\theta)$

110. $\cos(\theta) - \sec(\theta) = -\tan(\theta)\sin(\theta)$

111. $\cos(\theta)(\tan(\theta) + \cot(\theta)) = \csc(\theta)$

112. $\sin(\theta)(\tan(\theta) + \cot(\theta)) = \sec(\theta)$

113. $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$

114. $\dfrac{1}{\sec(\theta)+1} + \dfrac{1}{\sec(\theta)-1} = 2\csc(\theta)\cot(\theta)$

115. $\dfrac{1}{\csc(\theta)+1} + \dfrac{1}{\csc(\theta)-1} = 2\sec(\theta)\tan(\theta)$

116. $\dfrac{1}{\csc(\theta)-\cot(\theta)} - \dfrac{1}{\csc(\theta)+\cot(\theta)} = 2\cot(\theta)$

117. $\dfrac{\cos(\theta)}{1-\tan(\theta)} + \dfrac{\sin(\theta)}{1-\cot(\theta)} = \sin(\theta) + \cos(\theta)$

118. $\dfrac{1}{\sec(\theta)+\tan(\theta)} = \sec(\theta) - \tan(\theta)$

119. $\dfrac{1}{\sec(\theta)-\tan(\theta)} = \sec(\theta) + \tan(\theta)$

120. $\dfrac{1}{\csc(\theta)-\cot(\theta)} = \csc(\theta) + \cot(\theta)$

121. $\dfrac{1}{\csc(\theta)+\cot(\theta)} = \csc(\theta) - \cot(\theta)$

122. $\dfrac{1}{1-\sin(\theta)} = \sec^2(\theta) + \sec(\theta)\tan(\theta)$

123. $\dfrac{1}{1+\sin(\theta)} = \sec^2(\theta) - \sec(\theta)\tan(\theta)$

124. $\dfrac{1}{1-\cos(\theta)} = \csc^2(\theta) + \csc(\theta)\cot(\theta)$

125. $\dfrac{1}{1+\cos(\theta)} = \csc^2(\theta) - \csc(\theta)\cot(\theta)$

126. $\dfrac{\cos(\theta)}{1+\sin(\theta)} = \dfrac{1-\sin(\theta)}{\cos(\theta)}$

127. $\csc(\theta) - \cot(\theta) = \dfrac{\sin(\theta)}{1+\cos(\theta)}$

128. $\dfrac{1-\sin(\theta)}{1+\sin(\theta)} = (\sec(\theta) - \tan(\theta))^2$

In Exercises 129 - 132, verify the identity. You may need to review of the properties of absolute value and logarithms before proceeding.

129. $\ln|\sec(\theta)| = -\ln|\cos(\theta)|$

130. $-\ln|\csc(\theta)| = \ln|\sin(\theta)|$

131. $-\ln|\sec(\theta) - \tan(\theta)| = \ln|\sec(\theta) + \tan(\theta)|$

132. $-\ln|\csc(\theta) + \cot(\theta)| = \ln|\csc(\theta) - \cot(\theta)|$