

Worksheet 17 KEY - Trigonometric Identities (§7.2 and §7.3)

7. $\cos(75^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$

8. $\sec(165^\circ) = -\frac{4}{\sqrt{2} + \sqrt{6}} = \sqrt{2} - \sqrt{6}$

9. $\sin(105^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$

10. $\csc(195^\circ) = \frac{4}{\sqrt{2} - \sqrt{6}} = -(\sqrt{2} + \sqrt{6})$

11. $\cot(255^\circ) = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$

12. $\tan(375^\circ) = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = 2 - \sqrt{3}$

13. $\cos\left(\frac{13\pi}{12}\right) = -\frac{\sqrt{6} + \sqrt{2}}{4}$

14. $\sin\left(\frac{11\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$

15. $\tan\left(\frac{13\pi}{12}\right) = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = 2 - \sqrt{3}$

16. $\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$

17. $\tan\left(\frac{17\pi}{12}\right) = 2 + \sqrt{3}$

18. $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$

19. $\cot\left(\frac{11\pi}{12}\right) = -(2 + \sqrt{3})$

20. $\csc\left(\frac{5\pi}{12}\right) = \sqrt{6} - \sqrt{2}$

21. $\sec\left(-\frac{\pi}{12}\right) = \sqrt{6} - \sqrt{2}$

22. (a) $\cos(\alpha + \beta) = -\frac{\sqrt{2}}{10}$

(b) $\sin(\alpha + \beta) = \frac{7\sqrt{2}}{10}$

(c) $\tan(\alpha + \beta) = -7$

(d) $\cos(\alpha - \beta) = -\frac{\sqrt{2}}{2}$

(e) $\sin(\alpha - \beta) = \frac{\sqrt{2}}{2}$

(f) $\tan(\alpha - \beta) = -1$

23. (a) $\cos(\alpha + \beta) = -\frac{4 + 7\sqrt{2}}{30}$

(b) $\sin(\alpha + \beta) = \frac{28 - \sqrt{2}}{30}$

(c) $\tan(\alpha + \beta) = \frac{-28 + \sqrt{2}}{4 + 7\sqrt{2}} = \frac{63 - 100\sqrt{2}}{41}$

(d) $\cos(\alpha - \beta) = \frac{-4 + 7\sqrt{2}}{30}$

(e) $\sin(\alpha - \beta) = -\frac{28 + \sqrt{2}}{30}$

(f) $\tan(\alpha - \beta) = \frac{28 + \sqrt{2}}{4 - 7\sqrt{2}} = -\frac{63 + 100\sqrt{2}}{41}$

24. (a) $\sin(\alpha + \beta) = \frac{16}{65}$

(b) $\cos(\alpha - \beta) = \frac{33}{65}$

(c) $\tan(\alpha - \beta) = \frac{56}{33}$

$$25. \quad (a) \csc(\alpha - \beta) = -\frac{5}{4} \qquad (b) \sec(\alpha + \beta) = \frac{125}{117} \qquad (c) \cot(\alpha + \beta) = \frac{117}{44}$$

$$39. \cos(75^\circ) = \frac{\sqrt{2 - \sqrt{3}}}{2} \qquad 40. \sin(105^\circ) = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$41. \cos(67.5^\circ) = \frac{\sqrt{2 - \sqrt{2}}}{2} \qquad 42. \sin(157.5^\circ) = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$43. \tan(112.5^\circ) = -\sqrt{\frac{2 + \sqrt{2}}{2 - \sqrt{2}}} = -1 - \sqrt{2} \qquad 44. \cos\left(\frac{7\pi}{12}\right) = -\frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$45. \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2 - \sqrt{3}}}{2} \qquad 46. \cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$47. \sin\left(\frac{5\pi}{8}\right) = \frac{\sqrt{2 + \sqrt{2}}}{2} \qquad 48. \tan\left(\frac{7\pi}{8}\right) = -\sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}} = 1 - \sqrt{2}$$

$$49. \quad \bullet \sin(2\theta) = -\frac{336}{625} \qquad \bullet \cos(2\theta) = \frac{527}{625} \qquad \bullet \tan(2\theta) = -\frac{336}{527}$$

$$\bullet \sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{2}}{10} \qquad \bullet \cos\left(\frac{\theta}{2}\right) = -\frac{7\sqrt{2}}{10} \qquad \bullet \tan\left(\frac{\theta}{2}\right) = -\frac{1}{7}$$

$$50. \quad \bullet \sin(2\theta) = \frac{2520}{2809} \qquad \bullet \cos(2\theta) = -\frac{1241}{2809} \qquad \bullet \tan(2\theta) = -\frac{2520}{1241}$$

$$\bullet \sin\left(\frac{\theta}{2}\right) = \frac{5\sqrt{106}}{106} \qquad \bullet \cos\left(\frac{\theta}{2}\right) = \frac{9\sqrt{106}}{106} \qquad \bullet \tan\left(\frac{\theta}{2}\right) = \frac{5}{9}$$

$$51. \quad \bullet \sin(2\theta) = \frac{120}{169} \qquad \bullet \cos(2\theta) = -\frac{119}{169} \qquad \bullet \tan(2\theta) = -\frac{120}{119}$$

$$\bullet \sin\left(\frac{\theta}{2}\right) = \frac{3\sqrt{13}}{13} \qquad \bullet \cos\left(\frac{\theta}{2}\right) = -\frac{2\sqrt{13}}{13} \qquad \bullet \tan\left(\frac{\theta}{2}\right) = -\frac{3}{2}$$

$$52. \quad \bullet \sin(2\theta) = -\frac{\sqrt{15}}{8} \qquad \bullet \cos(2\theta) = \frac{7}{8} \qquad \bullet \tan(2\theta) = -\frac{\sqrt{15}}{7}$$

$$\bullet \sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{8 + 2\sqrt{15}}}{4} \qquad \bullet \cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{8 - 2\sqrt{15}}}{4} \qquad \bullet \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}}}$$

$$\tan\left(\frac{\theta}{2}\right) = 4 + \sqrt{15}$$

$$53. \quad \bullet \sin(2\theta) = \frac{24}{25} \qquad \bullet \cos(2\theta) = -\frac{7}{25} \qquad \bullet \tan(2\theta) = -\frac{24}{7}$$

$$\bullet \sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{5}}{5} \qquad \bullet \cos\left(\frac{\theta}{2}\right) = \frac{2\sqrt{5}}{5} \qquad \bullet \tan\left(\frac{\theta}{2}\right) = \frac{1}{2}$$

54. • $\sin(2\theta) = \frac{24}{25}$ • $\cos(2\theta) = -\frac{7}{25}$ • $\tan(2\theta) = -\frac{24}{7}$
 • $\sin\left(\frac{\theta}{2}\right) = \frac{2\sqrt{5}}{5}$ • $\cos\left(\frac{\theta}{2}\right) = -\frac{\sqrt{5}}{5}$ • $\tan\left(\frac{\theta}{2}\right) = -2$
55. • $\sin(2\theta) = -\frac{120}{169}$ • $\cos(2\theta) = \frac{119}{169}$ • $\tan(2\theta) = -\frac{120}{119}$
 • $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{26}}{26}$ • $\cos\left(\frac{\theta}{2}\right) = -\frac{5\sqrt{26}}{26}$ • $\tan\left(\frac{\theta}{2}\right) = -\frac{1}{5}$
56. • $\sin(2\theta) = -\frac{120}{169}$ • $\cos(2\theta) = \frac{119}{169}$ • $\tan(2\theta) = -\frac{120}{119}$
 • $\sin\left(\frac{\theta}{2}\right) = \frac{5\sqrt{26}}{26}$ • $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{26}}{26}$ • $\tan\left(\frac{\theta}{2}\right) = 5$
57. • $\sin(2\theta) = -\frac{4}{5}$ • $\cos(2\theta) = -\frac{3}{5}$ • $\tan(2\theta) = \frac{4}{3}$
 • $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{50 - 10\sqrt{5}}}{10}$ • $\cos\left(\frac{\theta}{2}\right) = -\frac{\sqrt{50 + 10\sqrt{5}}}{10}$ • $\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{5 - \sqrt{5}}{5 + \sqrt{5}}}$
 $\tan\left(\frac{\theta}{2}\right) = \frac{5 - 5\sqrt{5}}{10}$
58. • $\sin(2\theta) = -\frac{4}{5}$ • $\cos(2\theta) = -\frac{3}{5}$ • $\tan(2\theta) = \frac{4}{3}$
 • $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{50 + 10\sqrt{5}}}{10}$ • $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{50 - 10\sqrt{5}}}{10}$ • $\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{5 + \sqrt{5}}{5 - \sqrt{5}}}$
 $\tan\left(\frac{\theta}{2}\right) = \frac{5 + 5\sqrt{5}}{10}$
74. $\frac{\cos(2\theta) + \cos(8\theta)}{2}$ 75. $\frac{\cos(5\theta) - \cos(9\theta)}{2}$ 76. $\frac{\sin(8\theta) + \sin(10\theta)}{2}$
77. $\frac{\cos(4\theta) + \cos(8\theta)}{2}$ 78. $\frac{\cos(\theta) - \cos(5\theta)}{2}$ 79. $\frac{\sin(2\theta) + \sin(4\theta)}{2}$
80. $2 \cos(4\theta) \cos(\theta)$ 81. $-2 \cos\left(\frac{9}{2}\theta\right) \sin\left(\frac{5}{2}\theta\right)$ 82. $2 \sin\left(\frac{11}{2}\theta\right) \sin\left(\frac{1}{2}\theta\right)$
83. $2 \cos(4\theta) \sin(5\theta)$ 84. $\sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right)$ 85. $-\sqrt{2} \sin\left(\theta - \frac{\pi}{4}\right)$
90. $1 - \frac{x^2}{2}$ 91. $\frac{14x}{x^2 + 49}$ 92. $\ln|x + \sqrt{x^2 + 16}| - \ln(4)$