

Worksheet 19 KEY - Trigonometric Equations (§7.5)

1. $x = \frac{\pi k}{5}; x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}$
2. $x = \frac{\pi}{9} + \frac{2\pi k}{3}$ or $x = \frac{5\pi}{9} + \frac{2\pi k}{3}; x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$
3. $x = \frac{2\pi}{3} + \pi k$ or $x = \frac{5\pi}{6} + \pi k; x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$
4. $x = \frac{\pi}{24} + \frac{\pi k}{6}; x = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{3\pi}{8}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{7\pi}{8}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{11\pi}{8}, \frac{37\pi}{24}, \frac{41\pi}{24}, \frac{15\pi}{8}$
5. $x = \frac{3\pi}{8} + \frac{\pi k}{2}; x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$
6. $x = \frac{\pi}{12} + \frac{2\pi k}{3}$ or $x = \frac{7\pi}{12} + \frac{2\pi k}{3}; x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$
7. $x = \frac{\pi}{3} + \frac{\pi k}{2}; x = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$
8. No solution
9. $x = \frac{3\pi}{4} + 6\pi k$ or $x = \frac{9\pi}{4} + 6\pi k; x = \frac{3\pi}{4}$
10. $x = -\frac{\pi}{3} + \pi k; x = \frac{2\pi}{3}, \frac{5\pi}{3}$
11. $x = \frac{3\pi}{4} + \pi k$ or $x = \frac{13\pi}{12} + \pi k; x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{7\pi}{4}$
12. $x = -\frac{19\pi}{12} + 2\pi k$ or $x = \frac{\pi}{12} + 2\pi k; x = \frac{\pi}{12}, \frac{5\pi}{12}$
13. No solution
14. $x = \frac{5\pi}{8} + \frac{\pi k}{2}; x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
15. $x = \frac{\pi}{3} + \pi k$ or $x = \frac{2\pi}{3} + \pi k; x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
16. $x = \frac{\pi}{6} + \pi k$ or $x = \frac{5\pi}{6} + \pi k; x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
17. $x = \frac{\pi}{4} + \frac{\pi k}{2}; x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
18. $x = \frac{\pi}{3} + \pi k$ or $x = \frac{2\pi}{3} + \pi k; x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

19. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

21. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

23. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

25. $x = \frac{2\pi}{3}, \frac{4\pi}{3}, \arccos\left(\frac{1}{3}\right), 2\pi - \arccos\left(\frac{1}{3}\right)$

27. $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \arcsin\left(\frac{1}{3}\right), \pi - \arcsin\left(\frac{1}{3}\right)$

29. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

31. $x = \arctan(2), \pi + \arctan(2)$

33. $x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

35. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

37. $x = \frac{\pi}{3}, \frac{5\pi}{3}$

39. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

41. $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

43. $x = 0, \frac{\pi}{7}, \frac{2\pi}{7}, \frac{3\pi}{7}, \frac{4\pi}{7}, \frac{5\pi}{7}, \frac{6\pi}{7}, \pi, \frac{8\pi}{7}, \frac{9\pi}{7}, \frac{10\pi}{7}, \frac{11\pi}{7}, \frac{12\pi}{7}, \frac{13\pi}{7}$

44. $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

46. $x = \frac{\pi}{48}, \frac{11\pi}{48}, \frac{13\pi}{48}, \frac{23\pi}{48}, \frac{25\pi}{48}, \frac{35\pi}{48}, \frac{37\pi}{48}, \frac{47\pi}{48}, \frac{49\pi}{48}, \frac{59\pi}{48}, \frac{61\pi}{48}, \frac{71\pi}{48}, \frac{73\pi}{48}, \frac{83\pi}{48}, \frac{85\pi}{48}, \frac{95\pi}{48}$

47. $x = 0, \frac{\pi}{2}$

49. $x = \frac{\pi}{12}, \frac{17\pi}{12}$

51. $x = \frac{17\pi}{24}, \frac{41\pi}{24}, \frac{23\pi}{24}, \frac{47\pi}{24}$

20. $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

22. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

24. $x = \frac{\pi}{3}, \frac{5\pi}{3}$

26. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

28. $x = \frac{3\pi}{4}, \frac{7\pi}{4}, \arctan\left(\frac{1}{2}\right), \pi + \arctan\left(\frac{1}{2}\right)$

30. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

32. $x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

34. $x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{6}$

36. $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

38. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

40. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

42. No solution

45. $x = 0$

48. $x = \frac{\pi}{2}, \frac{11\pi}{6}$

50. $x = 0, \pi, \frac{\pi}{3}, \frac{4\pi}{3}$

52. $x = \frac{\pi}{6}, \frac{5\pi}{18}, \frac{5\pi}{6}, \frac{17\pi}{18}, \frac{3\pi}{2}, \frac{29\pi}{18}$

53. $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

54. $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

55. $x = 0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \pi, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

56. $x = \frac{\pi}{7}, \frac{\pi}{3}, \frac{3\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}, \frac{11\pi}{7}, \frac{5\pi}{3}, \frac{13\pi}{7}$

57. $x = 0, \frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7}, \frac{8\pi}{7}, \frac{10\pi}{7}, \frac{12\pi}{7}, \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

58. $x = \arcsin\left(\frac{-1 + \sqrt{5}}{2}\right) \approx 0.6662, \pi - \arcsin\left(\frac{-1 + \sqrt{5}}{2}\right) \approx 2.4754$

59. $x = -\frac{1}{2}$

60. $x = -1$

61. $x = \frac{2}{3}$

62. $x = -\frac{\sqrt{3}}{2}$

63. $x = 2\sqrt{2}$

64. $x = 6$

65. $x = \pm \frac{\sqrt{3}}{2}$

66. $x = \frac{1}{2}$

67. $x = -1, 0$

68. $x = -\sqrt{3}$

69. $[\pi, 2\pi]$

70. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right) \cup \left[\frac{4\pi}{3}, \frac{3\pi}{2}\right)$

71. $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$

72. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$

73. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$

74. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right]$

75. $\left(0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \pi\right) \cup \left(\pi, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right)$

76. $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$

77. No solution

78. $[0, 2\pi]$

79. $\left[0, \frac{\pi}{4}\right] \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left[\frac{7\pi}{4}, 2\pi\right]$

80. $[\operatorname{arccot}(4), \pi) \cup [\pi + \operatorname{arccot}(4), 2\pi)$