

Worksheet 4 KEY – Operations on Functions (§3.5)

1.

(a) $(f + g)(-3) = 2$

(d) $(g - f)(3) = 3$

(b) $(f - g)(2) = 3$

(e) $\left(\frac{f}{g}\right)(-2)$ is undefined

(c) $(fg)(-1) = 0$

(f) $\left(\frac{g}{f}\right)(3) = -2$

2.

(a) $(f + g)(1) = 5$

(d) $(fg)(4) = 0$

(b) $(f - g)(2) = 0$

(e) $\left(\frac{f}{g}\right)(1) = \frac{2}{3}$

(c) $(g - f)(3) = -3$

(f) $\left(\frac{g}{f}\right)(4)$ is undefined

3.

(a) $(f + g)(2) = 10, (f - g)(-1) = -9, (fg)\left(\frac{1}{2}\right) = -\frac{47}{16}, \left(\frac{f}{g}\right)(0) = 0$

(b) $(f + g)(2) = 3 + \sqrt{5}, (f - g)(-1) = 3 + \sqrt{2}, (fg)\left(\frac{1}{2}\right) = 0, \left(\frac{f}{g}\right)(0) = -\sqrt{3}$

(c) $(f + g)(2) = \frac{21}{5}, (f - g)(-1) = -1, (fg)\left(\frac{1}{2}\right) = \frac{1}{2}, \left(\frac{f}{g}\right)(0) = 0$

(d) $(f + g)(2) = \frac{17}{4}, (f - g)(-1) = 0, (fg)\left(\frac{1}{2}\right) = 1, \left(\frac{f}{g}\right)(0)$ is undefined

4.

(a) $(f + g)(x) = x^2 + 3x - 1, (f - g)(x) = x^2 - 3x + 1, (fg)(x) = 3x^3 - x^2,$
 $\left(\frac{f}{g}\right)(x) = \frac{x^2}{3x-1}$

(b) $(f + g)(x) = x^2 + 3x + 2, (f - g)(x) = x^2 - 3x - 10, (fg)(x) = 3x^3 +$
 $6x^2 - 12x - 24, \left(\frac{f}{g}\right)(x) = \frac{x-2}{3}$

(c) $(f + g)(x) = \frac{x^2+4}{2x}, (f - g)(x) = \frac{x^2-4}{2x}, (fg)(x) = 1, \left(\frac{f}{g}\right)(x) = \frac{x^2}{4}$

(d) $(f + g)(x) = x + \sqrt{x+1}, (f - g)(x) = x - \sqrt{x+1}, (fg)(x) = x\sqrt{x+1},$
 $\left(\frac{f}{g}\right)(x) = \frac{x}{\sqrt{x+1}}$

5.

(a) $(f \circ g)(3) = 4$

(c) $(g \circ f)(3) = -4$

(e) $(g \circ f \circ g)(0) = -3$

(b) $(f \circ f)(0) = 3$

(d) $(g \circ g)(-2) = 0$

(f) $f\left(f\left(f\left(f(f(1))\right)\right)\right) = 3$

6.

(a) $(g \circ f)(1) = 3$

(c) $(g \circ f)(2) = 0$

(e) $(f \circ f)(1) = 3$

(b) $(f \circ g)(3) = 4$

(d) $(f \circ g)(0) = 4$

(f) $(g \circ g)(1) = 0$

7.

$$(a) (g \circ f)(0) = 4, (f \circ g)\left(\frac{1}{2}\right) = \frac{5}{2}, (f \circ f)(-2) = -26$$

$$(b) (g \circ f)(0) = \sqrt{5}, (f \circ g)\left(\frac{1}{2}\right) = 5 + 2\sqrt{2}, (f \circ f)(-2) = -7$$

$$(c) (g \circ f)(0) = 24, (f \circ g)\left(\frac{1}{2}\right) = \frac{27-2\sqrt{42}}{8}, (f \circ f)(-2) = -14$$

$$(d) (g \circ f)(0) = \frac{6}{5}, (f \circ g)\left(\frac{1}{2}\right) = -5, (f \circ f)(-2) \text{ is undefined}$$

8.

$$(a) (g \circ f)(x) = 4x^2 + 12x, (f \circ g)(x) = 2x^2 - 15, (f \circ f)(x) = 4x + 9$$

$$(b) (g \circ f)(x) = |x^2 - 4|, (f \circ g)(x) = x^2 - 4, (f \circ f)(x) = x^4 - 8x^2 + 12$$

$$(c) (g \circ f)(x) = \sqrt{4 - x^2}, (f \circ g)(x) = 2 - x, (f \circ f)(x) = -x^4 + 6x^2 - 6$$

$$(d) (g \circ f)(x) = \frac{4x+1}{x}, (f \circ g)(x) = \frac{2x+1}{5x+2}, (f \circ f)(x) = \frac{x}{4x+1}$$